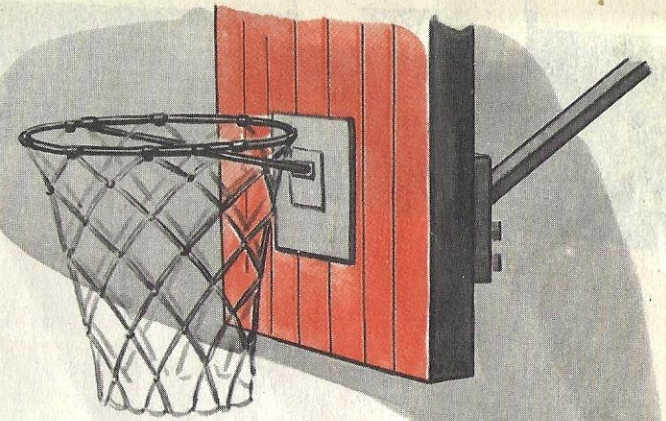




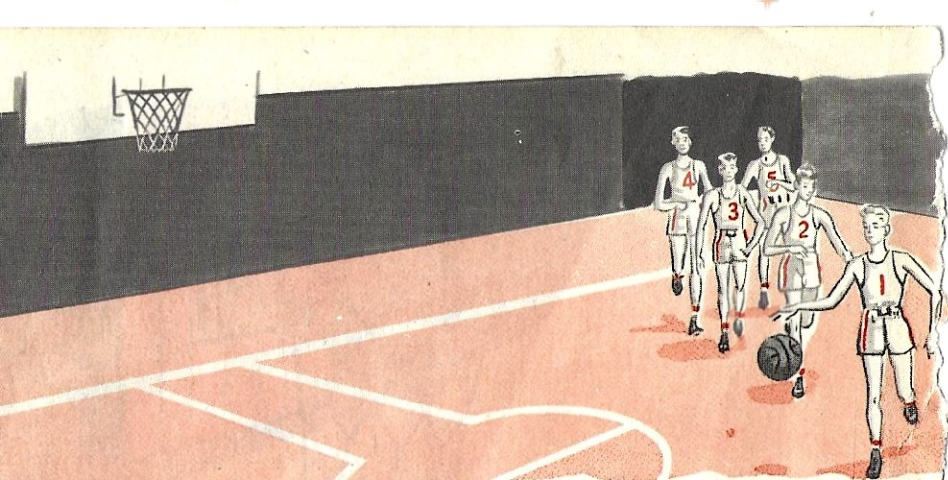
Cage ★ ★ ★ ★ Champions ★ ★ ★ ★

Adventures with numbers



Cage Champions





Cage

Champions

Adventures with numbers

JESSE OSBORN, Mathematics Consultant
Saint Louis Public Schools

ADELINE RIEFLING, Educational Counselor and
Reading Specialist, Saint Louis Public Schools

Illustrated by SALLY CAMANA

★ ★ ★ ★ ★ ★ ★

Webster Publishing Company

SAINT LOUIS • ATLANTA • DALLAS • LOS ANGELES



Adventures with numbers.

IN ORDER OF INCREASING DIFFICULTY

BUSY BEAVERS

RANGE RIDERS

STRAIGHT SHOOTERS

AIRPLANE ACES

HOME RUN HITTERS

CAGE CHAMPIONS

• • •

Copyright 1948 by

WEBSTER PUBLISHING COMPANY

Copyright in Great Britain and in the British Dominions and Possessions

All rights reserved

Printed in U. S. A.

★ Preface

Plan of Series. Adventures with Numbers is unique in organization. Each book has a motif taken from the pupils' world and captivating to the pupil. The motif furnishes the book's title. Young learners assume the role of Busy Beavers or Range Riders as they experience adventures with numbers. The boy or girl who is a little older plays that he is on the rifle range or piloting a plane as he gains further skill and understanding. The upper grade pupil takes part in a make-believe ball game, the rules and the scores of which provide motivation and sliding measures of achievement. Emphasis is placed on crediting the pupil with what he does correctly, without undermining his confidence when he fails to make a perfect total score, or the hypothetical 100%.

Unit Organization. Each book is divided into twelve units, each unit a little book in itself, with presentation of new material, practice in computation, and helps in problem solving. Pages of races, drills, and problems, with like captions and illustrations, occur in each unit of a book. The four closing pages of a unit are:

- a. Test on material of unit, with a goal.
- b. Helps for those who failed to reach goal a.
- c. Challenges for those who did reach goal a.
- d. Final unit test, geared to average pupil.

The first and seventh units of each book are given largely to reviewing and reteaching. Thus, twice a year, there are systematic campaigns to strengthen foundations so that the pupil can go into new work with new confidence, even though there have been gaps in his earlier work.

Non-Discriminating Differentials. Careful provisions are made for pupils with different learning rates. This is done in such a way as not to foster a superior attitude on the part of the fast learner nor to embarrass the one who learns slowly. Among the beavers, one who mends the wall can take as much pride in his work as the one who builds it higher. In basketball, one who recovers the ball is as valuable to the team as one who passes the ball. There is no higher nor lower with the pupil who does his best with the work that is appropriate for him. In Adventures with Numbers, there is differentiation without discrimination.

Individualized Program. Directions are addressed to pupils. Pupils are asked to choose partners, or to divide into groups or teams to do specific tasks.

They mark each other's papers and go on to the next task. This builds up a feeling of responsibility on the part of the pupil, and helps to release the teacher from constant direction of the class so that she can give help where it is most needed. The range between the slow learner and the fast learner in the average class is great. This series provides material and a plan so that the teacher can guide each pupil along at his own rate, rather than merely "covering" the material judged appropriate for the average pupil.

Readiness. Experience and experiments show that pupils do not become ready of themselves. We also know that teaching is wasted when the pupil is not ready. A program of preparing the pupil for each next step is skillfully woven into Adventures with Numbers. The «times-and» facts precede carrying in multiplication; station numbers precede uneven division. A boy's bicycle repair shop grows into a partnership and into a small corporation before stocks and bonds are presented. No process or idea is introduced until the pupil has been conditioned for it.

Presentation of New Concepts. Meanings, skills, and processes that are most difficult for the child to grasp are usually the ones that are most difficult to present clearly in a text. An author is tempted to be satisfied with something that he knows is inadequate, and to hope that teacher and pupil can fill in what is lacking. In Adventures with Numbers, no effort has been spared to present each new idea clearly and adequately. A pupil is not stopped today by the height of the ultimate goal in arithmetic, but by some little point in today's lesson that is not clear to him. Our presentations make the steps short and leave no gaps.

Using Arithmetic. A pupil may excel in book computation or book problems, and fail to see that the purpose of what he learns in school is to help him outside of school. He fails to carry what he learns in the little world within the school to the big world on the outside. Adventures with Numbers guides the pupil to the use of arithmetic in his everyday affairs, and helps him see arithmetic as a part of his whole self and not just a part of his school self.

Importance of Arithmetic. Quantitative thinking plays a more decisive role in our age than ever before. This is an age of number, measurement, science, and technology. The pupil who leaves school deficient in arithmetic is blocked from entering many fields. He will be handicapped in his efforts to earn a good living, or to buy most with what he does earn. For the individual who will carry his load and for the nation, arithmetic is truly an important part of the education for survival.

Contents

Unit VIII. Pages 183-208

Algebraic representation. Solving equations. Positive and negative numbers. Evaluating formulas. Line graphs.

Unit IX. Pages 209-234

Measurement of rectangular solids, prisms, cylinders, pyramids, cones, and spheres. Problem solving.

Unit X. Pages 235-260

Property insurance including rates, risks, protection, and policies. Automobile insurance. Problem solving.

Unit XI. Pages 261-286

Kinds and cost of life insurance. Health and accident insurance. Taxes. Problem solving.

Unit XII. Pages 287-313

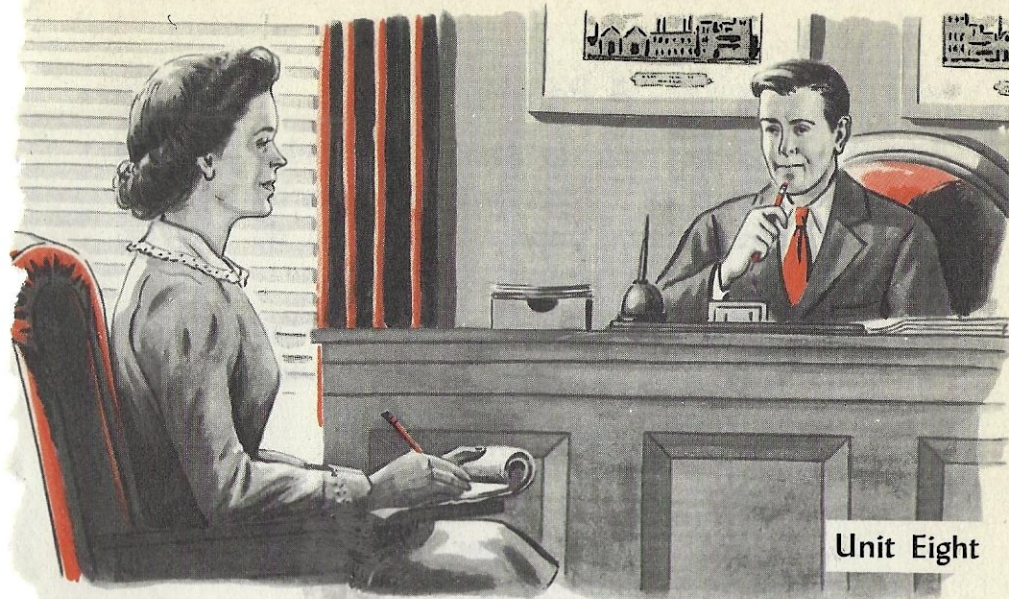
Ratio and proportion. Indirect measurement. Pythagorean formula. Squares and square roots. An integrated unit in which a group of pupils earn money to finance a trip.

Tables of Weights and Measures. Pages 314-315
Extra Practice. Pages 316-340

CONTINUING FROM

PART ONE

OF THIS BOOK



Unit Eight

Algebraic Shorthand

Did you ever watch a secretary take a letter in shorthand? Strange symbols appear on the page. They mean nothing to you, but to the stenographer they are a short-form language telling exactly what is to be written in the letter.

In algebra you learn a kind of shorthand that is useful in many lines of work. You have used some of this in formulas and simple equations, such as $A = l \times w$, and $5 + ? = 9$.

In algebra you make much use of letters to stand for numbers. *Six times a number increased by 7* is written in algebraic shorthand as $6n + 7$. Notice that the multiplication sign is not written between the 6 and the n . This way of writing products is the usual way in algebra.

A Partner Page

One partner will do Section A, while the other does Section B. They will then exchange and mark each other's papers. Both will do any example that either missed.

Section A. Write in algebraic shorthand.

1. A number increased by 7. **Answer:** $n+7$
2. A number decreased by 4.
3. Twice a number.
4. Half a number.
5. Seven times a number.
6. One fourth a number.
7. Three times a number decreased by 8.
8. Six times a number increased by 5.
9. One third a number increased by 2.
10. One fifth a number decreased by 1.

Section B. Write in algebraic shorthand.

1. A number decreased by 3. **Answer:** $n-3$
2. A number increased by 1.
3. One third of a number.
4. Four times a number.
5. Three times a number decreased by 5.
6. Seven times a number increased by 2.
7. One fourth a number increased by 1.
8. Half a number decreased by 4.
9. 8 increased by half a number.
10. Two thirds a number.

Using Letters in Making Equations

Four times a number plus 3 equals 15 may be written in algebraic shorthand as, $4x+3=15$.

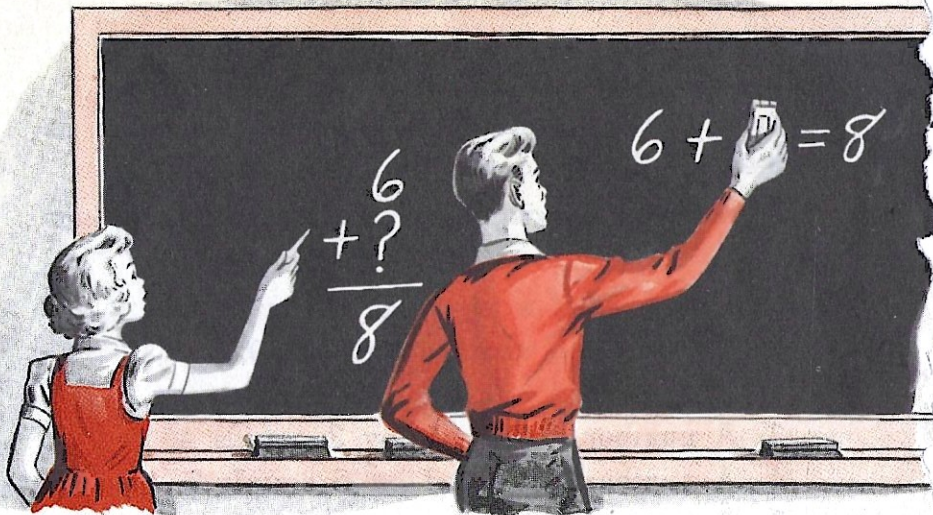
Work with a partner in writing the equations below.

Section A. Let x represent the number.

1. Five times a number equals 20.
2. One third a number equals 6.
3. A number decreased by 3 equals 4.
4. A number increased by 6 equals 9.
5. Twice a number increased by 4 is 10.
6. Half a number decreased by 3 is 4.
7. Seven times a number decreased by 1 is 20.
8. Six times a number increased by 2 is 20.
9. 6 plus four times a number is 14.
10. Two thirds a number is 12.

Section B. Let y represent the number.

1. Four times a number equals 28.
2. One fifth a number equals 12.
3. A number increased by 7 equals 10.
4. A number decreased by 4 equals 18.
5. Three times a number decreased by 2 is 46.
6. One third a number increased by 5 is 17.
7. 3 increased by three times a number is 45.
8. 40 decreased by three times a number is 4.
9. 4 plus nine times a number is 22.
10. Three fourths a number is 12.



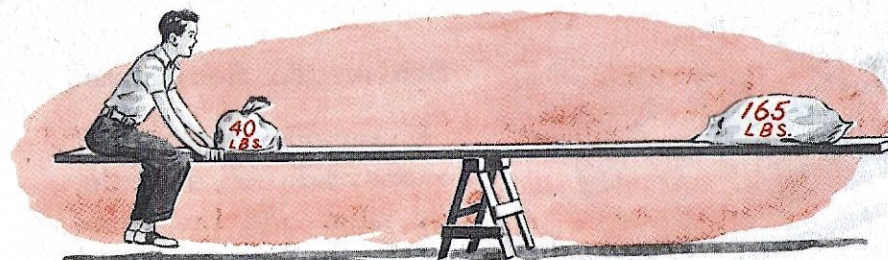
Becoming Better Acquainted with Equations

An equation is a statement of fact. It contains definite numbers or letters that stand for definite numbers. An equation must contain an equals sign. The equals sign is the basis for the name **equation**. An equation is a balanced statement, because the two sides are equal.

When the numbers in an equation are small, you can find the value of a letter without knowing any rules of algebra.

Find the value of the letter in each of these equations.

- | | | |
|-----------------------|--------------------|--------------------|
| 1. $5 + n = 9$ | $3 + x = 5$ | $1 + y = 8$ |
| 2. $x + 3 = 8$ | $s + 2 = 8$ | $u + 5 = 7$ |
| 3. $3n = 15$ | $7k = 14$ | $8t = 24$ |
| 4. $8 - x = 6$ | $9 - y = 5$ | $5 - n = 3$ |
| 5. $\frac{1}{2}x = 5$ | $\frac{1}{3}y = 7$ | $\frac{1}{4}x = 3$ |



Balancing Equations

On this page you will learn how to keep an equation in balance, just as you would keep a seesaw in balance.

1. Ted and a 40-pound bag of sand exactly balance a 165-pound bag of sand on the other side of the seesaw. How much does Ted weigh?

To have Ted alone on the left side you must remove the 40-pound bag of sand.

Then you must remove 40 pounds of sand from the right side to keep the balance.

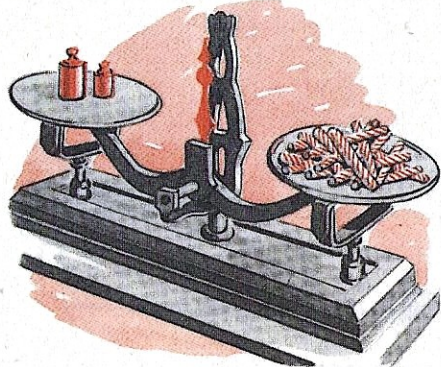
$$x + 40 - 40 = 165 - 40, \text{ or } x = 125$$

What number must you take away from the left side of each of these equations to have x alone on the left?

- | | | |
|----------------|-------------|-------------|
| 2. $x + 5 = 9$ | $x + 1 = 9$ | $x + 3 = 7$ |
| 3. $x + 7 = 8$ | $x + 4 = 8$ | $x + 2 = 9$ |
| 4. $x + 3 = 9$ | $x + 1 = 3$ | $x + 5 = 7$ |
| 5. $x + 9 = 9$ | $x + 6 = 8$ | $x + 4 = 7$ |

What number must you take away from the right side of each equation above to keep the balance?

Solve each equation above for x .



Scales and Equations

If something is added to one side of a scale, you must add the same weight to the other side *to keep the balance*.

When you take something from one side of a scale, you must take the same weight from the other side *to keep the balance*.

These ideas are true for equations also. When you do something to one side, you must do the same to the other side *to keep the two sides equal*.

On page 187, you saw that 40 had been added to x , and that x and 40 made 165, or $x + 40 = 165$. To find x , you had to take away a 40 from each side.

If 20 is taken from each side of the equation $x = 70$, you have the equation $x - 20 = 50$. To find x when $x - 20 = 50$ is given, you must put back the 20 on each side.

$$x - 20 + 20 = 50 + 20, \text{ or } x = 70.$$

In each equation below, what number must you put back on the left side to have the full value of x ? What number, then, must you add to the right side to keep the balance? Solve for x .

- | | | |
|----------------|-------------|-------------|
| 1. $x - 1 = 7$ | $x - 3 = 2$ | $x - 4 = 5$ |
| 2. $x - 2 = 6$ | $x - 5 = 3$ | $x - 6 = 2$ |
| 3. $x - 7 = 1$ | $x - 4 = 4$ | $x - 5 = 2$ |
| 4. $x - 3 = 6$ | $x - 2 = 7$ | $x - 6 = 1$ |

Using the Multiplication Rule

1. I am thinking of a number. Half of this number is 7. What would you do to find the number? What is the number?

$$\begin{aligned} \frac{1}{2}x &= 7 \\ 2x \left(\frac{1}{2}x \right) &= 2 \times 7 \\ x &= 14 \end{aligned}$$

Notice that both sides of this equation were treated alike. The left side had to be multiplied by 2, for 2 times $\frac{1}{2}x$ is x . The right side, then, had to be multiplied by 2.

2. What would you multiply each of the following by to have just x ?

$$\frac{1}{2}x \quad \frac{1}{6}x \quad \frac{1}{4}x \quad \frac{1}{8}x \quad \frac{1}{3}x \quad \frac{1}{5}x$$

In each equation below, x stands for a number. What would you multiply each side of the equation by to find the number?

- | | | |
|---------------------------------|------------------------------|------------------------------|
| 3. $\frac{1}{2}x = 3$ | $\frac{1}{3}x = 5$ | $\frac{1}{4}x = 2$ |
| 4. $\frac{1}{5}x = 4$ | $\frac{1}{6}x = 2$ | $\frac{1}{8}x = 3$ |
| 5. $\frac{1}{2}x = 5$ | $\frac{1}{3}x = 6$ | $\frac{1}{6}x = 1$ |
| 6. $\frac{1}{3}x = \frac{2}{3}$ | $\frac{1}{4}x = \frac{3}{4}$ | $\frac{1}{8}x = \frac{1}{4}$ |

7. Solve for x in each equation above.

To check an answer, write the value you find for the letter in place of the letter in the given equation. If the statement that comes from this is true, the answer checks. The answer, $x = 14$, in the example at the top of the page checks, for $\frac{1}{2}$ of $14 = 7$.

8. Check the answers for Equations 3 - 6.

Using the Division Rule

$$3x = 72$$

$$\frac{3x}{3} = \frac{72}{3}$$

$$x = 24$$

1. There are 72 pieces of candy in three equal boxes. How many pieces are in one box?

What must you do to find the answer?

The answer, $x = 24$, checks, for $3 \times 24 = 72$.

Notice that both sides of the equation were treated alike. The left side had to be divided by 3 to have x alone. The right side had to be divided by 3 to keep the balance.

2. What do you divide each of the following by to have just x ?

$$3x \quad 7x \quad 9x \quad 4x \quad 8x \quad 5x \quad 2x \quad 6x$$

Solve for x and check.

$$3. \quad 2x = 12 \quad 5x = 15 \quad 9x = 18$$

$$4. \quad 3x = 21 \quad 4x = 32 \quad 2x = 16$$

$$5. \quad 6x = 48 \quad 8x = 32 \quad 7x = 35$$

$$6. \quad 7x = 14 \quad 2x = 14 \quad 5x = 30$$

The first thing to do in solving any equation is to decide what must be done to the left side to have x alone. Then, do this same thing to both the left side and the right side.

What do you do to solve each of the following?

Solve these equations.

$$7. \quad x + 5 = 7 \quad x - 4 = 3 \quad \frac{1}{4}x = 2$$

$$8. \quad 5x = 35 \quad x + 3 = 7 \quad x - 2 = 6$$

$$9. \quad \frac{1}{3}x = 5 \quad 6x = 42 \quad x + 1 = 8$$

FIRST STEP

$$\frac{1}{3}x - 4 = 3$$

$$\frac{1}{3}x - 4 + 4 = 3 + 4$$

$$\frac{1}{3}x = 7$$

SECOND STEP

$$3 \times \frac{1}{3}x = 3 \times 7$$

$$x = 21$$

Two-Step Equations

Miss Grace began two-step equations like this:

Don't tell me your age. I want to find it, but you will have to give me some clues.

Mabel, multiply your age by 3, add 2, and give me the number. That will be my clue. I will write it like this, $3x + 2 = 38$. I can solve this equation in two steps, and then I will have your age.

First Step

$$3x + 2 = 38$$

$$3x + 2 - 2 = 38 - 2$$

$$3x = 36$$

Second Step

$$\frac{3x}{3} = \frac{36}{3}$$

$$x = 12$$

This answer checks, for $3 \times 12 + 2 = 38$.

Show each first step and second step, and solve these two-step equations. Check.

$$1. \quad 3x + 4 = 28 \quad 5x + 1 = 36 \quad 2x + 7 = 25$$

$$2. \quad 4x - 3 = 25 \quad 7x - 2 = 40 \quad 6x - 8 = 40$$

$$3. \quad \frac{1}{3}x + 5 = 11 \quad \frac{1}{6}x + 9 = 12 \quad \frac{1}{8}x + 5 = 10$$

$$4. \quad \frac{1}{2}x - 3 = 12 \quad \frac{1}{3}x - 7 = 2 \quad \frac{1}{4}x - 1 = 5$$

When you solve equations like those above, the first step is to add or to subtract. The second step is to multiply or to divide. Treat both sides of the equation alike.

A Search for Missing Numbers

Solving an equation is solving a mystery. It is finding a missing number. The clues that are given will lead you to the number.

This is a "partner page." Which of you can be the better Sherlock Holmes?

Section A. Solve for x and check.

- | | | |
|-------------|------------------|--------------------|
| 1. $x+1=4$ | $2x=6$ | $2x+1=5$ |
| 2. $x-3=6$ | $\frac{1}{3}x=4$ | $3x-1=8$ |
| 3. $x+2=6$ | $3x=6$ | $2x+5=11$ |
| 4. $x-1=3$ | $\frac{1}{2}x=3$ | $4x-3=1$ |
| 5. $x+3=5$ | $4x=16$ | $7x+1=15$ |
| 6. $x-5=4$ | $\frac{1}{4}x=5$ | $\frac{1}{2}x-4=2$ |
| 7. $x+5=7$ | $5x=15$ | $\frac{1}{5}x+1=5$ |
| 8. $x-3=8$ | $\frac{1}{6}x=2$ | $\frac{1}{3}x-3=2$ |
| 9. $x+4=4$ | $6x=24$ | $\frac{1}{6}x-1=7$ |
| 10. $x-4=4$ | $\frac{1}{5}x=6$ | $\frac{1}{4}x-2=4$ |

Section B. Solve for x and check.

- | | | |
|-------------|------------------|--------------------|
| 1. $x-2=5$ | $3x=12$ | $5x-3=7$ |
| 2. $x+3=7$ | $\frac{1}{2}x=5$ | $3x+1=7$ |
| 3. $x-5=2$ | $5x=30$ | $7x-4=24$ |
| 4. $x+6=8$ | $\frac{1}{3}x=6$ | $2x+5=19$ |
| 5. $x-4=3$ | $7x=42$ | $4x-7=17$ |
| 6. $x+4=9$ | $\frac{1}{4}x=2$ | $\frac{1}{4}x+3=8$ |
| 7. $x-1=5$ | $4x=24$ | $\frac{1}{3}x-2=5$ |
| 8. $x+5=8$ | $\frac{1}{5}x=4$ | $\frac{1}{2}x+1=9$ |
| 9. $x-3=9$ | $6x=48$ | $\frac{1}{5}x-5=3$ |
| 10. $x+2=2$ | $\frac{1}{6}x=3$ | $\frac{1}{8}x+6=8$ |

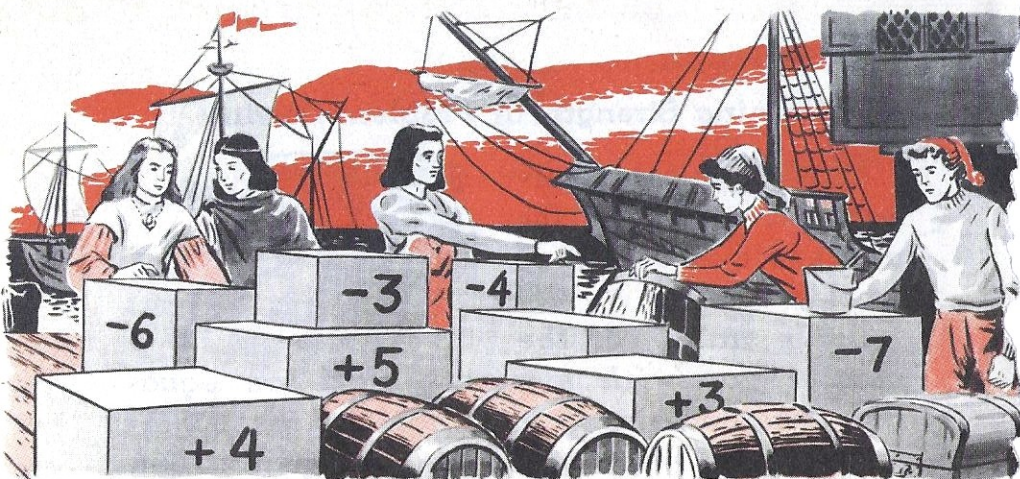
Gaining Strength in Problem Solving

By these simple problems you can learn the algebraic method of problem solving.



Let x stand for the number you wish to find. Make an equation that contains x and that fits the conditions of the problem. Solve the equation and check the answer.

1. My age plus 10 equals 22. How old am I?
2. My age minus 4 equals 9. How old am I?
3. A half dozen eggs cost 28¢. What is the cost of a dozen eggs?
4. Three pounds of bacon cost 96¢. What is the cost per pound?
5. One fourth the number of pages in a book plus 12 is equal to 100. How many pages are in the book?
6. The number of girls was 3 less than twice the number of boys. There were 25 girls. How many boys were there?
7. The number of boys was 3 more than half the number of girls. There were 15 boys. How many girls were there?
8. Multiply my age by 7 and subtract 5. This makes 100. How old am I?
9. Divide my father's age by 7 and add 5. This makes 10. How old is my father?



Old Time Cargoes

Merchants of Flanders had ordered boxes of dates and figs from Syria. This was 500 years ago. Each box was expected to weigh a certain number of pounds, but the actual weight of most of them was more or less than this expected weight.

The weigher marked $+4$ on a box whose actual weight was 4 pounds more than its expected weight and -6 on a box whose actual weight was 6 pounds less than its expected weight.

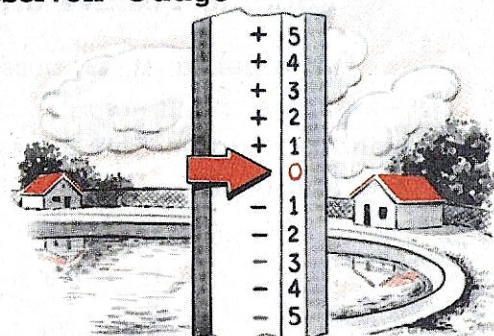
1. One merchant got boxes marked $+5$, -3 , -4 . Was the total actual weight of the three boxes more or less than their total expected weight?

2. Another merchant got two boxes marked -7 and $+3$. How should a third box be marked so that the total actual weight of the three boxes will exactly equal their total expected weight?

Reading the Reservoir Gauge

This is the gauge that tells how much water is in the reservoir. When the arrow points to 0, the water supply is considered **normal**. When the arrow points to $+2$, the water level is 2 in.

above normal, and when the arrow points to -3 , the water level is 3 in. **below** normal.



1. The arrow pointed to -3 one day and to $+2$ the next. How many inches had the water level risen?

2. The arrow pointed to $+4$ one day and to -3 the next. How many inches had the water level fallen?

3. On Monday the arrow pointed to -1 . During the week the water level rose 5 inches. Where did the arrow point then?

4. One day the arrow pointed to $+2$. During that week the water level dropped 6 inches. Where did the arrow point then?

A number with a plus sign before it is a **positive number**.

A number with a minus sign before it is a **negative number**.

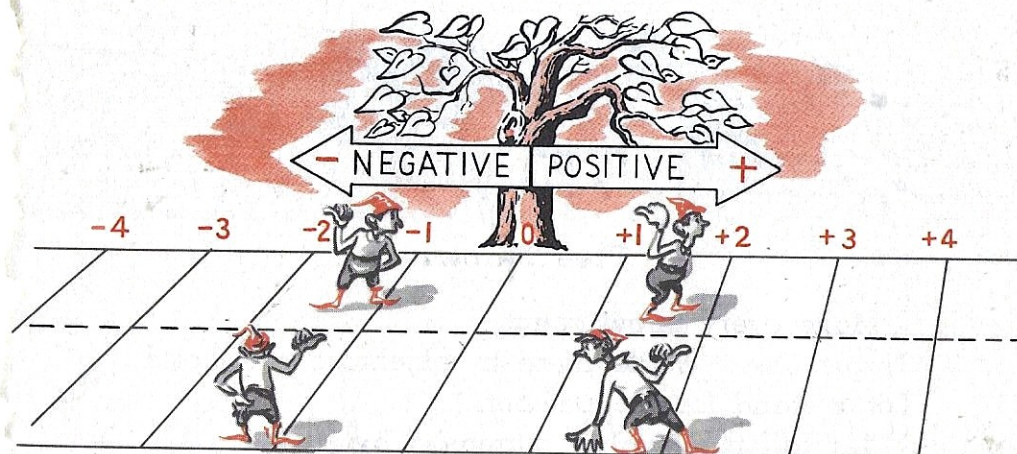
Adding Positive and Negative Numbers

In algebra it is possible to add positive and negative numbers. To understand this, one may think of gains and losses. Add a gain of \$5 and a loss of \$3. What is the sum?

$$\begin{array}{rcl}
 \text{Gain} & \$5 & +\$5 \\
 \text{Add Loss } \$3 & \text{or } + & -\$3 \\
 \hline
 \text{Net Gain} & \$2 & +\$2
 \end{array}$$

Write in the form indicated above and solve.

1. Add a gain of \$6 and a loss of \$2.
2. Add a gain of \$2 and a loss of \$5.
3. Add a gain of \$3 and a gain of \$4.
4. Add a loss of \$3 and a loss of \$4.
5. The temperature rose 3° in the morning and 6° in the afternoon. What was the total rise?
6. A boy earned \$4 and spent \$9. What was the result?
7. A child walked up 5 steps and then walked down 5 steps. Where was he then?
8. A football team was pushed back 4 yards and then another 8 yards. Add its losses.
9. A football team lost 6 yards and gained 15 yards. What was its net gain?
10. A boy gained 12 pounds in a year. In a short illness he lost 8 pounds. What was the net gain?
11. A river rose 5 feet above flood level and then fell 6 feet. Where did it stand then?



Stepping Along Number Lane

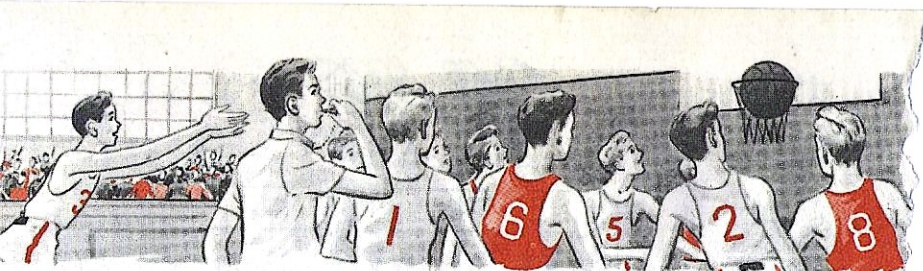
Start at 0. Take these steps. Tell where you are.

1. 3 steps to the right and 5 to the left.
2. 4 steps to the left and 6 to the right.
3. 1 step to the right and 2 to the right.
4. 2 steps to the left and 1 to the left.
5. 4 steps to the right and 1 to the left.

In each exercise above, write the steps as positive and negative numbers and find their sum.

Step along Number Lane and find the sums.

6. $\begin{array}{cccccccc} -1 & -2 & -3 & -4 & +1 & +2 & +3 & +4 \\ +3 & +4 & +2 & +1 & -3 & -1 & -3 & -2 \end{array}$
7. $\begin{array}{cccccccc} -1 & -1 & -3 & -2 & +1 & +1 & +3 & +2 \\ -2 & -1 & -1 & -2 & +2 & +1 & +1 & +2 \end{array}$
8. $\begin{array}{cccccccc} -1 & -2 & -4 & -2 & -4 & +1 & -3 & +2 \\ +4 & +4 & +4 & +3 & +3 & -2 & +4 & -2 \end{array}$



Free Throws

Make every throw count.

Throw One: Write these in algebraic shorthand.

Let x stand for the number.

- Half the number increased by 9.
- Three times the number decreased by 7.
- Four times the number plus 8 equals 40.
- One third the number minus 3 equals 8.

Throw Two: Solve for x and check.

- | | | |
|-----------------------|------------------|--------------------|
| 5. $x+7=10$ | $x+1=4$ | $2x+3=7$ |
| 6. $x-1=5$ | $3x-4=8$ | $x-9=2$ |
| 7. $8x=56$ | $\frac{1}{3}x=6$ | $\frac{1}{2}x-2=5$ |
| 8. $\frac{1}{4}x+1=7$ | $7x=42$ | $\frac{1}{6}x=12$ |

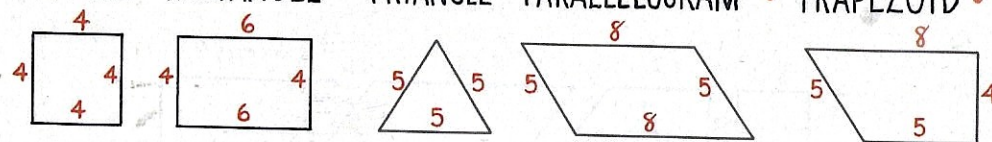
Throw Three: Find the values when $x=6$ and $y=4$.

- | | | |
|--------------------|---------------|----------------|
| 9. $2x$ | $3y$ | $\frac{1}{3}x$ |
| 10. $\frac{1}{2}y$ | $x+y$ | $x-y$ |
| 11. xy | $\frac{x}{y}$ | $\frac{y}{x}$ |
| 12. $2x-3y$ | $2xy$ | $4y-x$ |

Throw Four: Use the number scale on page 197 and find the sums.

- | | | |
|-------------------|---------------|---------------|
| 13. -4 and $+3$ | -2 and $+6$ | $+4$ and $+2$ |
| 14. $+3$ and $+2$ | -3 and $+5$ | -1 and $+2$ |
| 15. $+2$ and -1 | $+5$ and -1 | -3 and -2 |
| 16. -1 and -3 | $+3$ and -3 | -5 and $+6$ |

• SQUARE • RECTANGLE • TRIANGLE • PARALLELOGRAM • TRAPEZOID •

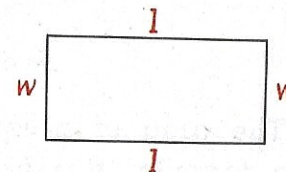


Perimeter Formulas

The **perimeter** of a figure is the distance around it.

1. Find the perimeter of each figure shown at the top of the page.

2. Show that the perimeter of the figure at the right may be written in these three ways:



$$p = l + w + l + w; \quad p = 2l + 2w; \quad p = 2(l + w).$$

These are formulas for the perimeter of a rectangle. A formula is a rule written in algebraic shorthand. The letters used in formulas are usually the initial letters of words. What words are represented by l , w , and p ?

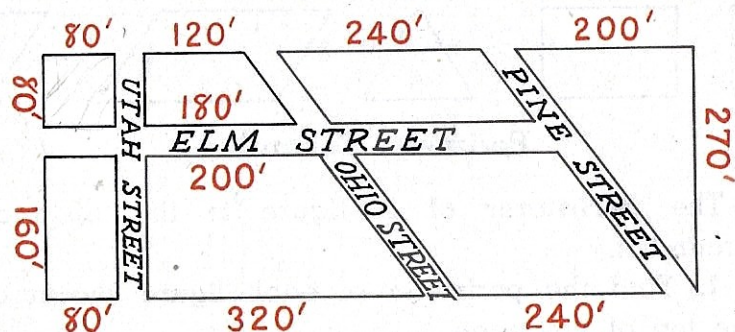
3. Use the formula $p = 2l + 2w$, and find p , when (a) $l=6$, $w=3$; (b) $l=7$, $w=5$.

4. The side of a square is s . Show that the formula for the perimeter may be written as

$$p = s + s + s + s, \text{ or as } p = 4s.$$

5. Use the formula $p = 4s$ and find p when (a) $s=5$; (b) $s=7$; (c) $s=8$.

6. The side of an equilateral triangle is s . Show that the formula for the perimeter may be written as $p = s + s + s$ or as $p = 3s$. Find p when $s=14$.



Area Formulas

The area of a square with side s is given by the formula $A = s^2$. Remember, s^2 means $s \times s$ and is read "s square."

Find the areas of the following squares:

1. $s = 5$
2. $s = 7$
3. $s = 9$

The area of a rectangle is given by the formula, $A = lw$. Remember, lw means $l \times w$.

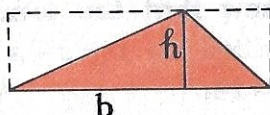
Find the areas of the following rectangles:

	4.	5.	6.	7.	8.
l	10"	16"	$22\frac{1}{2}"$	32.5'	6.75'
w	6"	5"	$6\frac{1}{2}"$	8'	4'

One dimension of a rectangle is often known as the base and the other as the height. In that case, the area is given by the formula $A = bh$.

9. Find the area of a rectangle whose base is 8 inches and whose height is 5 inches.

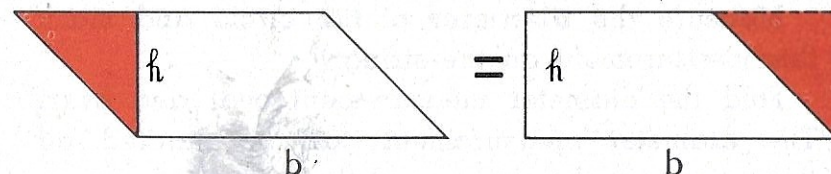
The area of a triangle is given by the formula $A = \frac{bh}{2}$.



Find the areas of the following triangles:

	10.	11.	12.	13.	14.
b	15"	28"	16'	7'	8.25'
h	8"	10"	6'	4'	8'

The area of a parallelogram is given by the formula $A = bh$.

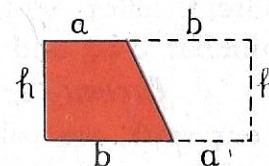


Find the areas of these parallelograms:

	15.	16.	17.	18.	19.
b	13"	17"	32'	17'	3.75'
h	7"	8"	6'	4'	8'

The area of a trapezoid is given by the formula $A = \frac{h(a+b)}{2}$.

Find the areas of these trapezoids:



	20.	21.	22.	23.	24.
a	4"	6"	3'	24'	18'
b	8"	7"	9'	32'	24'
h	3"	4"	5'	9'	7'

25. Write the formula for finding the area of each of the city blocks shown at the top of page 200. Use these formulas to find the area of each block.



Circle Formulas

1. Use a string to measure the **circumference** of a circle. A dinner plate or an oatmeal box will do. Cut the string to show this measurement.

Measure the **diameter** of the circle and mark this measurement on the string.

Fold the diameter measurement over and over. The diameter measurement can be marked off three times on the string, and there is a little string left over. The short length of string left over is about $\frac{1}{7}$ of the diameter. Approximately,

$$3\frac{1}{7} \times \text{diameter} = \text{circumference}$$

More exactly, this formula may be written with the Greek letter π , pronounced **pie**, in place of the number $3\frac{1}{7}$, and in this order,

$$\text{Circumference} = \pi \times \text{diameter}$$

In our work, we will use $3\frac{1}{7}$ or $\frac{22}{7}$ as the value of π . For more nearly accurate results, 3.1416 may be used for π .

2. Show by measuring and marking on the same string that the radius is half the diameter. Show the facts of these formulas with a string.

3. $d = 2r$

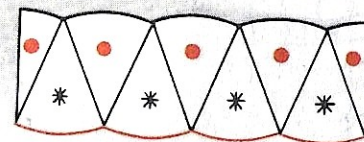
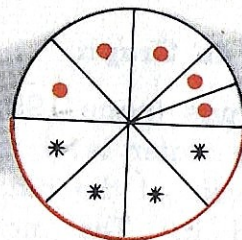
5. $c = \pi d$

4. $\frac{d}{2} = r$

6. $c = \pi \times 2r = 2\pi r$

7. Find d when $r=5$; when $r=3\frac{1}{2}$.

8. Find r when $d=8$; when $d=21$.



The Area of a Circle

A circle may be cut as a pie and the pieces arranged as in the figure above. This figure is much like a rectangle, with length equal to half the circumference and width equal to the radius. The formula for the area of the figure may be written $A = \frac{c}{2} \times r$. This formula also gives the area of the circle.

Find A when c and r are given as—

	1.	2.	3.	4.	5.	6.
c	22	44	66	88	11	33
r	3	7	10	14	1	5

Another formula may be used for finding the area of a circle when only the radius is given. This formula is $A = \pi r^2$. If you wish, you may study the equations at the right and discover how this formula is obtained.

$$A = \frac{c}{2} \times r$$

$$\text{and } c = 2\pi r$$

$$A = \frac{2\pi r}{2} \times r$$

$$A = \pi r^2$$

Find A for each of these values of r .

7. $r = 3\frac{1}{2}$

$r = 7$

$r = 10\frac{1}{2}$

$r = 14$

8. $r = 1\frac{3}{4}$

$r = 5\frac{1}{4}$

$r = 8\frac{3}{4}$

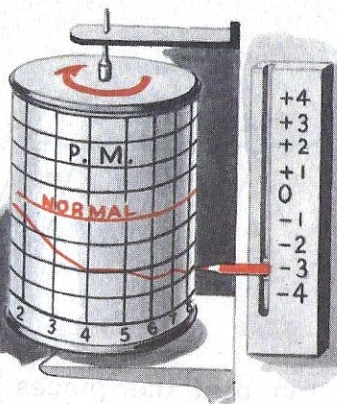
$r = 12\frac{1}{4}$

9. $r = 4$

$r = 6$

$r = 5$

$r = 9$



A Line Graph

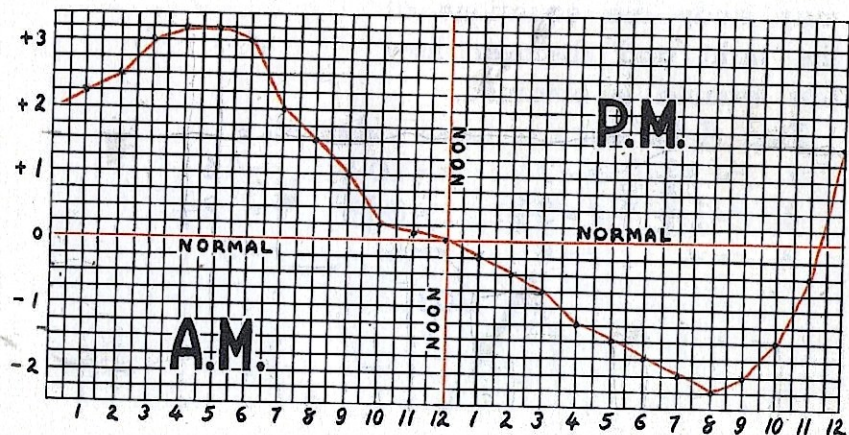
The gauge (page 195) shows the water level in the reservoir at the time you read it. The machine at the left keeps a continuous record of the height of the water in the

reservoir. As the water level rises or falls, the pencil moves up or down on the revolving cylinder to make a line graph. The record for one day is shown unrolled at the bottom of the page.

How many inches is the water level above or below normal at each of these hours? Give answers correct to the quarter inch.

- | | | | |
|-----------|---------|--------|-------------|
| 1. 1 A.M. | 7 A.M. | 1 P.M. | 7 P.M. |
| 2. 2 A.M. | 8 A.M. | 2 P.M. | 8 P.M. |
| 3. 3 A.M. | 9 A.M. | 3 P.M. | 9 P.M. |
| 4. 4 A.M. | 10 A.M. | 4 P.M. | 10 P.M. |
| 5. 5 A.M. | 11 A.M. | 5 P.M. | 11 P.M. |
| 6. 6 A.M. | 12 noon | 6 P.M. | 12 midnight |

7. Why is the water level higher at some hours than at other hours?



The Tip-Off

Write in algebraic shorthand.

- Twice a number decreased by 9.
- Two thirds a number increased by 6.
- Three times a number increased by 1 is 25.
- One fourth of a number minus 3 equals 6.

Solve the equations.

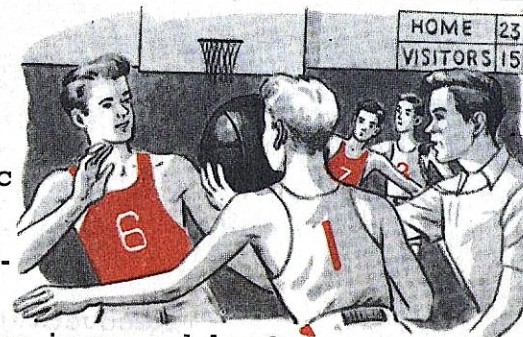
- | | |
|------------------------|----------------------------|
| 5. $x - 6 = 4$ | 9. $5x - 1 = 24$ |
| 6. $\frac{1}{4}x = 12$ | 10. $x + 7 = 9$ |
| 7. $6x = 12$ | 11. $\frac{1}{3}x - 2 = 7$ |
| 8. $2x + 3 = 7$ | 12. $\frac{1}{2}x + 1 = 5$ |

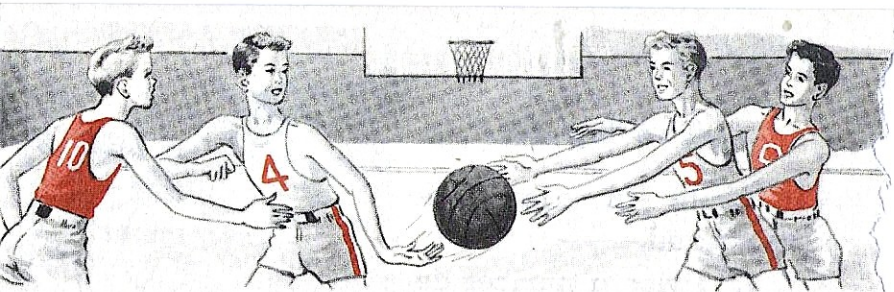
Which red number checks in the equation before it?

- | | |
|--------------------------------------|-------------------------------------|
| 13. $5x - 3 = 38$ 7 | 15. $\frac{1}{2}x + 5 = 9$ 6 |
| 14. $\frac{1}{2}x - 1 = 5$ 12 | 16. $7x + 5 = 25$ 3 |
- In the formula $4s = p$, find s when $p = 24$.
 - In the formula $p = 2l + 2w$, find p when $l = 7$ and $w = 3$.
 - In the formula $A = lw$, find A when $l = 6$ and $w = 5$.
 - In the formula $lw = A$, find w when $l = 8$ and $A = 24$.

21. Divide my age by 2 and add 15. The answer will be 21. How old am I?

If you had 18 or more answers right, work on page 207.





Recovering the Ball

A. If your mistakes in the Tip-Off were in—

Problems	Work pages	Problems	Work pages
1, 2	184	8, 9, 11, 12	191
3, 4	185	10	187
5	186, 188	13-16	189
6	189	17-20	199, 200
7	190	21	198

B. Through carelessness, some pupils forget to use this important rule: *Whatever you do to one side of an equation, you must do to the other side, also.*

For each equation below, answer the questions:

a What must I do on the left side to find x ?

b What, then, must I do on the right side?

- $x+4=7$ $x-8=3$ $3x=9$
- $\frac{1}{2}x=4$ $2x+1=7$ $\frac{1}{3}x-3=5$

C. Other pupils confuse the multiplication rule and the division rule in solving equations.

To solve $\frac{1}{4}x=5$, you multiply both sides by 4.

To solve $4x=24$, you divide both sides by 4.

Say the rule you use and the answer.

- $3x=12$ $\frac{1}{3}x=6$ $2x=8$
- $\frac{1}{2}x=8$ $6x=12$ $\frac{1}{6}x=12$

Passing the Ball

This page is a peek in- to real high school algebra. Expressions like $4x$, $3x$, $6y$, or $7c$ are called **terms**. The numbers before the letters are called **coefficients**. The coefficient of $4x$ is 4, of w is 1, of $\frac{1}{2}a$ is $\frac{1}{2}$.

Terms having the same letter are **like terms**, and those with different letters are **unlike terms**.

The sum of 6 plums and 2 plums is 8 plums.

The sum of $6x$ and $2x$ is $8x$.

You add like terms in algebra by adding their coefficients.

1. Add the following.

$$\begin{array}{r} 6x \\ 5x \end{array} \quad \begin{array}{r} 5m \\ 3m \end{array} \quad \begin{array}{r} 7t \\ t \end{array} \quad \begin{array}{r} 2y \\ 8y \end{array} \quad \begin{array}{r} a \\ 6a \end{array} \quad \begin{array}{r} 4r \\ r \end{array} \quad \begin{array}{r} 15q \\ 38q \end{array}$$

2. Three times my age plus 4 times my age equals 91. How old am I? Use the equation $3x+4x=91$.

8 plums $-$ 3 plums $=$ 5 plums, and $8x-3x=5x$.

You subtract like terms in algebra by subtracting their coefficients.

3. Subtract $5t$ from $12t$; x from $7x$; $2y$ from $8y$.

The sum of $2x$ and $3y$ is written $2x+3y$. Since the terms are unlike, you cannot combine the coefficients. The difference of $2x$ and $3y$ is written $2x-3y$.



Goal for Unit Eight

The answers are in jumbled order below the heavy line. Pick the right ones.



Write the problem numbers: 1-18. After each number write the letter that identifies an answer that fits.

Write in algebraic shorthand.

1. A number increased by 4 equals 17.
2. A number decreased by 4 equals 17.
3. One third of a number plus 4 equals 17.
4. Three times a number plus 4 equals 17.

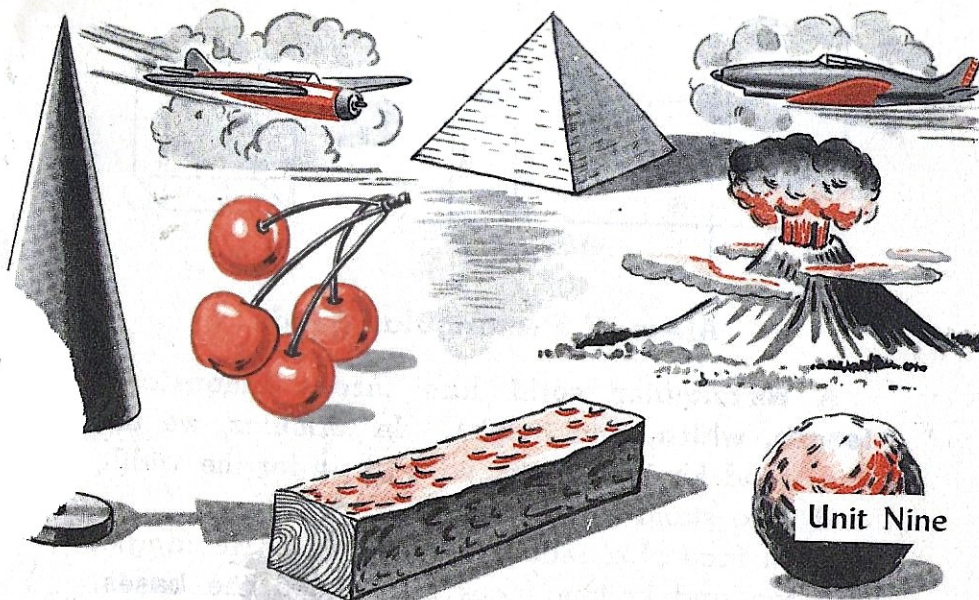
Solve the equations.

5. $3x=18$
6. $\frac{1}{3}x=6$
7. $x+2=5$
8. $2x-3=11$
9. $\frac{1}{4}x+5=9$
10. $x-1=1$
11. $4x-3=17$
12. $\frac{1}{2}x-3=9$

Find the values when $x=3$, $y=8$, $z=12$.

13. xz
14. $2z-3x$
15. $z-4x$
16. $\frac{z}{x}$
17. $y+z$
18. $3x-y$

- | | | | |
|------------------------|------|-------|-------|
| A. $\frac{1}{4}x=17$ | G. 0 | M. 6 | S. 18 |
| B. $3x+4=17$ | H. 1 | N. 7 | T. 20 |
| C. $\frac{1}{3}x+4=17$ | I. 2 | O. 8 | U. 24 |
| D. $x-4=17$ | J. 3 | P. 9 | V. 36 |
| E. $x+4=17$ | K. 4 | Q. 15 | W. 44 |
| F. $4x+1=17$ | L. 5 | R. 16 | X. 56 |



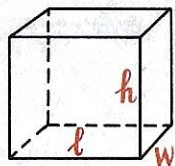
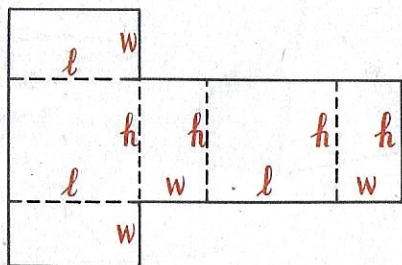
Unit Nine

Solids and Their Measures

In the long ago, man found ideas of form in the things about him. From the confusion of shapes in trees, stones, and mountains, he chose those most pleasing and useful to him. He chose wisely, for the prism, pyramid, cylinder, cone, and sphere have served him well through all the centuries and continue to serve well.

Today, also, man turns to nature for ideas of form. A study of birds and fish provides ideas for streamlining. However, use of new forms does not lessen the need for the old. Whatever the shape of the plane, the bearings in its motor are spheres, and its fuel comes in cylinders.

In this unit we study basic geometric forms.



Areas of Rectangular Solids

A rectangular solid has three dimensions: **length**, **width**, and **height**. In formulas, we use l to stand for the length, w to stand for the width, and h to stand for the height.

Each face of a rectangular solid is a rectangle. The top and bottom faces are called the **bases**. The other faces are called **lateral faces**.

In formulas, we let p stand for the **perimeter** of the base, and let B stand for the **area** of the base.

The sum of the areas of the lateral faces is called the **lateral area**. In formulas, we let L stand for the lateral area.

The lateral area plus the areas of the two bases is the **total area**. We let T stand for total area.

If the surface of a rectangular solid were laid out flat, it would appear as above at the left.

One way to find the lateral area is to find the area of each lateral face separately, and find their sum.

$$L = lh + wh + lh + wh = 2lh + 2wh$$

Another way to find the lateral area is to find the perimeter of the base and multiply this perimeter by the height of the rectangular solid.

$$p = 2l + 2w \text{ and } L = ph$$

The area of a base is given by the formula:

$$B = lw$$

The total area T of the rectangular solid is given by:

$$T = ph + 2B$$

1. Make a rectangular solid out of paper, and use it to prove the formulas above.

Find L and T in Problems 2-9.

2. $l = 15$ in., $w = 12$ in., $h = 8$ in.

3. $l = 10$ ft., $w = 8$ ft., $h = 3$ ft.

4. $l = 16$ yd., $w = 12$ yd., $h = 7$ yd.

Areas are found as units of square measure.

For example, if the linear units are inches, the area will be in square inches. Be sure that you name each answer correctly.

5. $l = 16$ in., $w = 14$ in., $h = 5$ in.

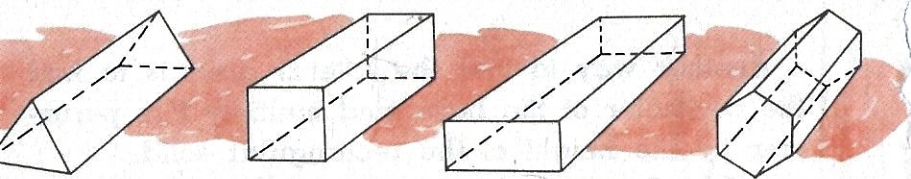
6. $l = 24$ ft., $w = 10$ ft., $h = 6$ ft.

7. $l = 13$ yd., $w = 7$ yd., $h = 4$ yd.

When the dimensions are given in different units of linear measure, they must be changed so that all measures will be given in the same unit of linear measure.

8. $l = 2$ ft., $w = 18$ in., $h = 8$ in. (2 ft. = 24 in.)

9. $l = 2$ ft., $w = 15$ in., $h = 1$ ft.



ANGULAR • SQUARE • RECTANGULAR • HEXAGONAL

Areas of Prisms

A **prism** has two equal and parallel bases. These bases may be triangles, squares, rectangles, or other figures bounded by straight lines. If the base of a prism is an equilateral triangle or a square, s will stand for the length of a side.

The area of a square is given by the formula: $B = s^2$

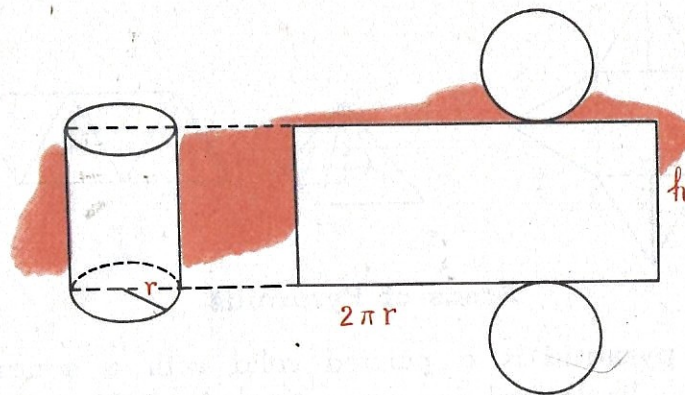
A rectangular solid is a rectangular prism.

The lateral area of any prism is given by the formula: $L = ph$

The total area of any prism is given by the formula: $T = ph + 2B$

Find the lateral area and the total area for these prisms using the formulas for L and T .

1. Square base with $s = 5$ in.; $h = 6$ in.
2. Square base with $s = 3\frac{1}{2}$ in.; $h = 3\frac{3}{4}$ in.
3. Square base with $s = 6.2'$; $h = 4.5'$.
4. Base a rectangle with $l = 3$ yd., $w = 2$ yd., and $h = 5$ yd.
5. Base a triangle with $B = 6$ sq. ft.; $p = 12$ ft.; $h = 8$ ft.
6. Make drawings showing lateral areas and total areas of prisms described in Ex. 1-5.



Areas of Cylinders

A **cylinder** has two equal and parallel circles as bases. Its lateral surface is curved. In formulas, r stands for the radius, d for the diameter, c for the circumference, and B for the area of a base of the cylinder. Use the figures above to show that these formulas are true:

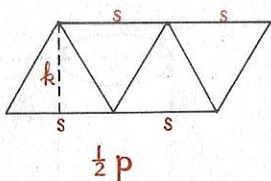
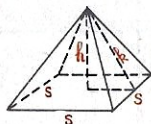
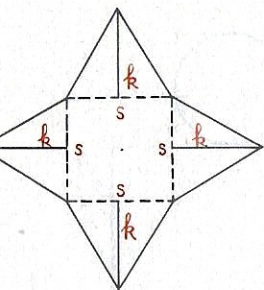
$$d = 2r, c = 2\pi r, \text{ and } B = \pi r^2$$

The lateral area and the total area of a cylinder are given by the formulas:

$$L = 2\pi rh, \text{ and } T = 2\pi rh + 2\pi r^2$$

Find L and T for these cylinders.

- | | |
|---------------------------|---------------------------|
| 1. $r = 4''$, $h = 5''$ | 7. $d = 6'$, $h = 3'$ |
| 2. $r = 7''$, $h = 4''$ | 8. $d = 14'$, $h = 2'$ |
| 3. $r = 2'$, $h = 6'$ | 9. $d = 21''$, $h = 5''$ |
| 4. $r = 21'$, $h = 2'$ | 10. $d = 7''$, $h = 3''$ |
| 5. $r = 5''$, $h = 4''$ | 11. $d = 35'$, $h = 4'$ |
| 6. $r = 8''$, $h = 10''$ | 12. $d = 21'$, $h = 3'$ |



Areas of Pyramids

A **pyramid** is a pointed solid with a square base. Its lateral faces are triangles. The point of the pyramid is called the **vertex**.

The perpendicular distance from the vertex to the base is the height of the pyramid. The altitude of a lateral face is the slant height of the pyramid. In formulas we let h stand for the height and k for the slant height.

The lateral area and the total area of a pyramid are given by the formulas:

$$L = \frac{1}{2} pk, \text{ and } T = \frac{1}{2} pk + B$$

1. The area of the base of a square pyramid is 25 sq. in., and the area of each lateral face is 15 sq. in. What is the lateral area? What is the total area?

Find L and T for these square pyramids.

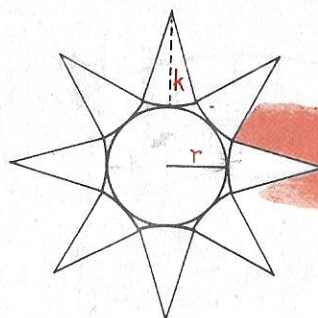
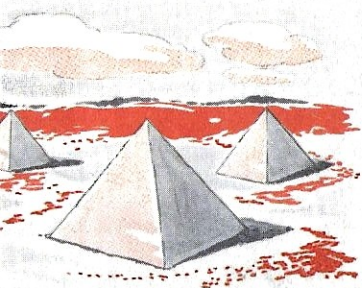
2. $s = 3''$, $k = 2''$

4. $s = 6'$, $k = 5'$

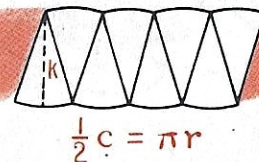
3. $s = 8'$, $k = 5'$

5. $s = 7''$, $k = 9''$

6. When built the Great Pyramid of Egypt had $s = 756$ ft. and $k = 610$ ft. Find L .



$$C = 2\pi r$$



Areas of Cones

A **cone** is a pointed solid with a circle for the base. You use the same plan for developing area formulas for the cone as you used for the pyramid.

The lateral area and the total area of the cone are given by the formulas:

$$L = \pi rk, \text{ and } T = \pi rk + \pi r^2$$

1. The lateral area of a cone is 220 sq. in. and the area of the base is 154 sq. in. What is the total area of the cone?

Find the lateral area and total area of each of these cones.

2. $r = 7''$, $k = 12''$

5. $d = 7''$, $k = 6''$

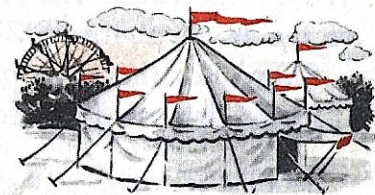
3. $r = 14'$, $k = 20'$

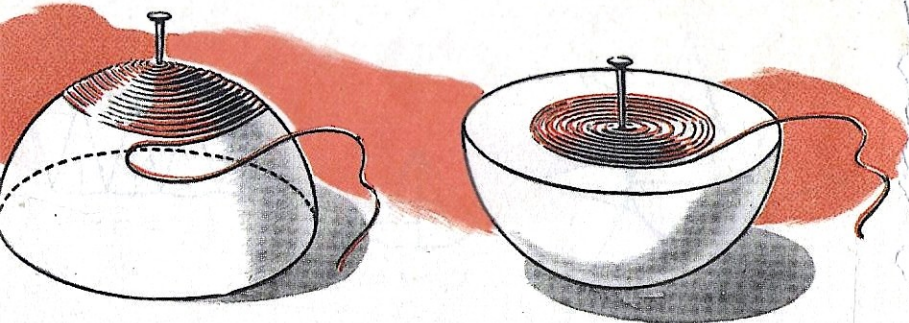
6. $d = 14'$, $k = 10'$

4. $r = 21''$, $k = 30''$

7. $d = 21''$, $k = 15''$

8. The diameter of a circular circus tent is 56 ft. The lower part is a cylinder 12 ft. high; the upper part is a cone with slant height of 30 ft. How many square yards of canvas were needed?





Areas of Spheres

The distance from the center of a sphere to any point on the surface is the radius of the sphere. A half sphere is called a **hemisphere**.

You can use a wooden hemisphere and heavy cord to find a formula for the area of a sphere.

A. Drive a nail at the center of the curved part of the hemisphere. Begin at the nail and wind the string around and around until the curved surface is completely covered. Unwind the string and measure its length.

B. Drive a nail at the center of the flat part of the hemisphere. Begin at the nail and wind the string until the flat surface is completely covered. Unwind the string and measure its length.

C. It takes twice as much string to cover the curved surface of the hemisphere as to cover the flat surface. The area of the flat surface is πr^2 . Why? Explain: The area of the curved surface of the hemisphere is $2\pi r^2$.

Show that the area of a sphere is given by the formula:

$$T = 4\pi r^2$$

Find the areas of these spheres.

1. $r = 14''$ 4. $d = 8'$ 7. $r = 3.5$ yd.
2. $d = 21''$ 5. $r = 5'$ 8. $d = 35$ yd.
3. $r = 35''$ 6. $d = 6'$ 9. $r = 10\frac{1}{2}$ yd.

10. The earth is approximately a sphere with average diameter of about 7,914 miles. What is the area of the earth's surface?

11. The moon is approximately a sphere with diameter of about 2,160 miles. What is the area of the moon's surface?

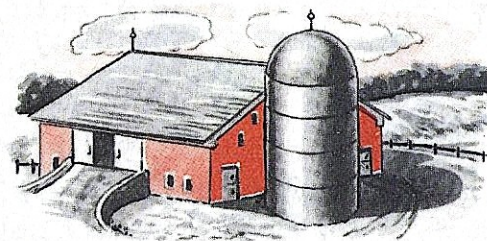
12. The diameter of a basketball is 10 inches. How many square inches of material are needed to cover it?

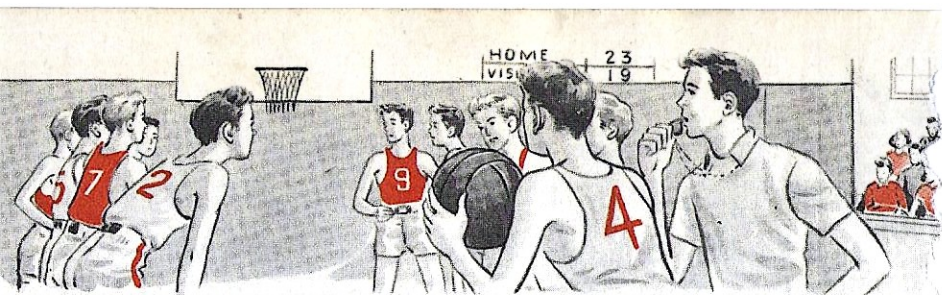
13. A beach ball is 28 inches in diameter. What is the area of its surface?

14. A spherical gas tank is 42 feet in diameter. What is the area of its surface?

15. Another spherical gas tank is 84 ft. in diameter. What is the area of its surface?

16. A cylindrical silo is 14 ft. in diameter and 20 ft. high. The roof of the silo is a hemisphere with diameter 14 ft. What is the area of the outside surface of the silo, including the roof?





Free Throws

Check answers after each "throw." Leave each answer in best form.

First Throw: Change.

1. 18 inches to feet and inches
2. 5 feet to yards and feet
3. 1 foot and 6 inches to inches
4. 1 yard and 1 foot to feet
5. Add.
1 yd. 2 ft. 6 in.
2 yd. 2 ft. 8 in.
6. Subtract.
5 yd. 2 ft. 6 in.
2 yd. 2 ft. 9 in.

Second Throw: Change.

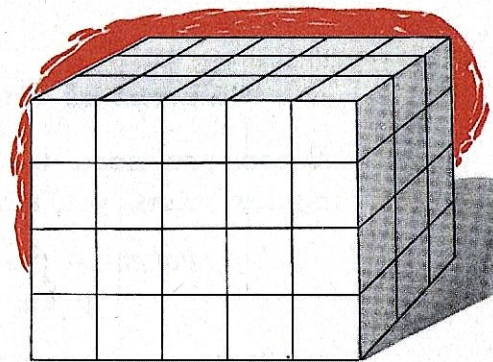
7. 160 sq. in. to sq. ft. and sq. in.
8. 14 sq. ft. to sq. yd. and sq. ft.
9. 1 sq. yd. and 4 sq. ft. to sq. ft.
10. 1 sq. ft. and 16 sq. in. to sq. in.
11. Add.
sq. yd. sq. ft. sq. in.
5 6 80
2 7 80
12. Subtract.
sq. yd. sq. ft. sq. in.
8 5 16
2 8 36

Third Throw: Supply the missing numbers.

13. ? gal. make 1 cu. ft.
14. ? cu. in. make 1 gal.
15. ? cu. in. make 1 cu. ft.
16. ? cu. ft. make 1 bu.
17. ? cu. in. make 1 bu.
18. ? cu. ft. make 1 cu. yd.

Volumes of Prisms

A child piles his blocks into a rectangular solid. If you should ask him how many blocks he has, he might tear down his building and count them. You have learned a better way. Let us review this better way.



There are 5 blocks in the front row of the top layer. There are 3 rows. This makes 15 blocks in all in the top layer.

There are 4 layers, with 15 blocks in each layer. This makes 60 blocks in the pile.

Suppose that each block is a cube 1 in. on a side. The volume of such a cube is 1 cu. in. The volume of the pile of blocks would be 60 cu. in. This volume can be found quickly, as

$$5 \times 3 \times 4 \text{ cu. in.} = 60 \text{ cu. in.}$$

When you need to find the volume of a rectangular solid, think:

$$\text{Volume} = \text{Length} \times \text{Width} \times \text{Height} = l \times w \times h$$

Find these volumes.

1. $l=9$, $w=6$, $h=5$
2. $l=8$, $w=2$, $h=6$
3. $l=6$, $w=2$, $h=4$
4. $l=7$, $w=5$, $h=3$
5. $l=3$, $w=2$, $h=8$
6. $l=4'$, $w=2'$, $h=6'$
7. $l=5''$, $w=3''$, $h=7''$
8. $l=6'$, $w=3'$, $h=8'$
9. $l=6''$, $w=3''$, $h=2''$
10. $l=8''$, $w=5''$, $h=3''$

Volumes of Prisms and Cylinders

When you need to find the volume of a rectangular prism, you may think:

$$\text{Volume} = \text{area of base} \times \text{height}$$

$$V = B \times h$$

1. Find the area of each base in Exercises 1-10 on page 219. Then, use the formula above to find each of the volumes.

The volume of any prism is given by the formula:
 $V = Bh$

2. The area of the base of a triangular prism is 28 sq. in. The height of the prism is 5 in. What is the volume of the prism in cubic inches?

Find the volumes of these prisms.

3. $B = 4$ sq. in., $h = 3$ in.

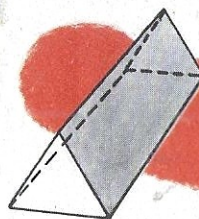
4. $B = 8$ sq. ft., $h = 6$ ft.

5. $B = 5$ sq. in., $h = 8$ in.

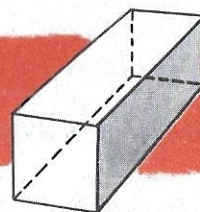
6. An ice-cream carton is a rectangular prism 4 in. by $3\frac{1}{2}$ in. by $2\frac{1}{8}$ in. Will the carton hold a pint? (231 cu. in. = 1 gal.)

7. The area of the base of a coal bin is 56 sq. ft. How many cubic feet of coal will it take to fill this bin to a height of 6 ft.?

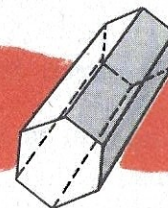
8. Allowing 42 cu. ft. of coal to a ton, how many tons will the bin of Problem 7 hold?



TRIANGULAR PRISM



SQUARE PRISM



HEXAGONAL PRISM



CYLINDER

When you need to find the volume of any prism or cylinder, think:

$$\text{Volume} = \text{Area of base} \times \text{height}$$

$$V = B \times h$$

Find the volumes of these cylinders.

9. $B = 4$ sq. ft., $h = 3$ ft.

10. $B = 8$ sq. in., $h = 6$ in.

11. $B = 5$ sq. ft., $h = 8$ ft.

The area of the base of a circular cylinder is given by the formula:

$$B = \pi r^2$$

The volume of a circular cylinder is given by the formula:

$$V = \pi r^2 h$$

Find the volumes of these cylinders.

12. $r = 6''$, $h = 7''$ 192

14. $d = 14'$, $h = 12'$

13. $r = 3\frac{1}{2}''$, $h = 4''$ 15. $d = 21'$, $h = 10'$

16. A cylindrical can is 7 in. in diameter and 6 in. high. What is its volume in cubic inches? Does it hold more or less than a gallon?

17. A cylindrical container is 18.5 in. in diameter and 8 in. high. What is its volume in cubic inches? Does it hold more or less than a bushel?

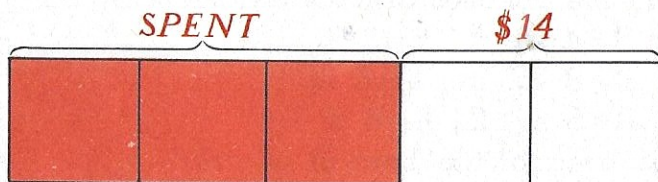
Gaining Strength in Problem Solving

Drawing a diagram is helpful in solving certain types of problems.

A. *One part and the fraction which that part is of the whole are given. You are asked to find the whole.*

1. Sam spent $\frac{3}{5}$ of his money and had \$14 left. How much did he have at first?

Draw a rectangle to stand for all of Sam's money. Divide the rectangle into 5 equal spaces, and shade 3 of them to represent the money



spent. The 2 unshaded spaces stand for the \$14 that was not spent.

Then, 1 space stands for \$7, and all 5 spaces stand for \$35.

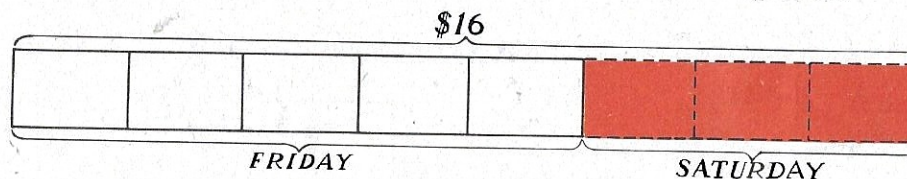
Check: $\frac{3}{5} \times \$35 = \21 , money spent.

$\$35 - \$21 = \$14$, money left.

B. *The whole and the fraction which one part is of the other part are given. You are asked to find the parts.*

2. Ross earned \$16 on Friday and Saturday. He earned $\frac{3}{5}$ as much on Saturday as on Friday. How much did he earn each day?

Draw a rectangle to stand for the money Ross earned on Friday. Divide it into 5 equal spaces.



Extend the length of the rectangle by 3 of the equal spaces to represent the money earned on Saturday.

Then 8 spaces stand for \$16, and 1 space stands for \$2.

His earnings on Friday were $5 \times \$2 = \10 .

His earnings on Saturday were $3 \times \$2 = \6 .

Check: $\$10 + \$6 = \$16$; $\frac{3}{5} \times \$10 = \6 .

Each problem below is either type A or type B. Decide which type it is, and then solve by use of a diagram.

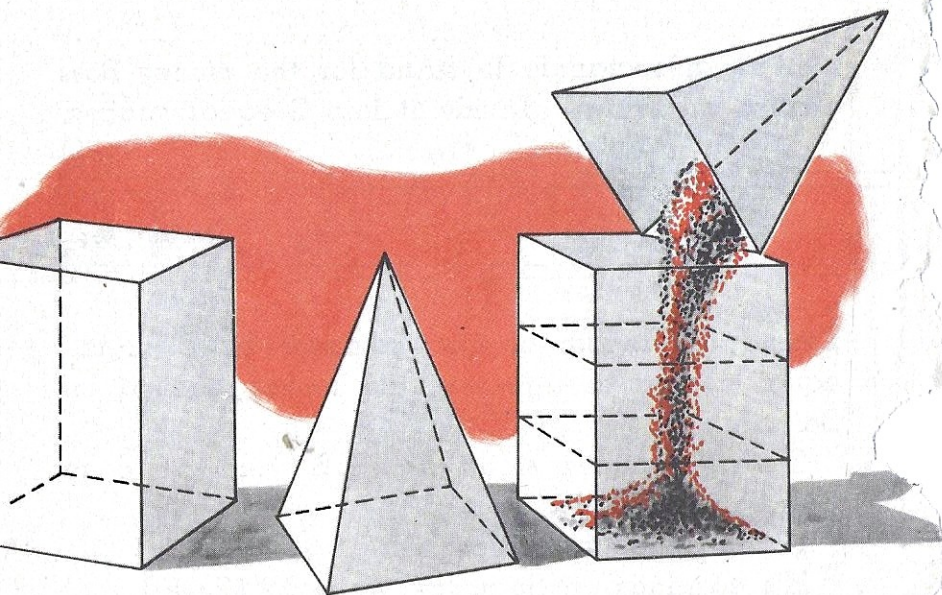
3. Eva spent $\frac{3}{8}$ of her money for shoes and the remainder for a hat. The hat cost \$15. How much money did she spend in all?

4. A ball and a bat cost \$6. The bat cost $\frac{2}{3}$ as much as the ball. How much did each cost?

5. Jack rode $\frac{5}{7}$ of the way and walked the remaining 10 mi. How long was the trip?

6. A book cost $\frac{3}{4}$ as much as a fountain pen. Together they cost \$14. How much did each cost?

7. A house and lot cost \$8400. The lot cost 40% as much as the house. How much did each cost?



Volume of a Pyramid

The base of the prism above is equal to the base of the pyramid and the height of the prism is equal to the height of the pyramid. Make models.

Use sand and show that the volume of the pyramid is equal to one third the volume of the prism.

The volume of the prism is given by the formula:

$$V = Bh$$

The volume of the pyramid is given by the formula:

$$V = \frac{Bh}{3}$$

1. Find the volume of a square pyramid, when a side of the base is 5 in. and the height is 6 in.

$$B = 5 \times 5 = 25$$

$$V = \frac{25 \times 6}{3} = 50$$

The volume is 50 cu. in.

2. A square prism and a square pyramid have their bases equal and their heights equal. The volume of the prism is 54 cu. in. What is the volume of the pyramid?

3. A square prism and a square pyramid have their bases and their heights equal. The volume of the pyramid is 48 cu. in. What is the volume of the prism?

4. The base of a pyramid is a square 6 in. on a side. The height of the pyramid is 5 in. What is its volume?

5. The base of a prism is a square 6 in. on a side. The height of the prism is 5 in. What is its volume?

Find the volumes of the pyramids.

6. $s=3$, $h=7$

9. $s=5$, $h=8$

7. $s=6$, $h=4$

10. $s=12$, $h=8$

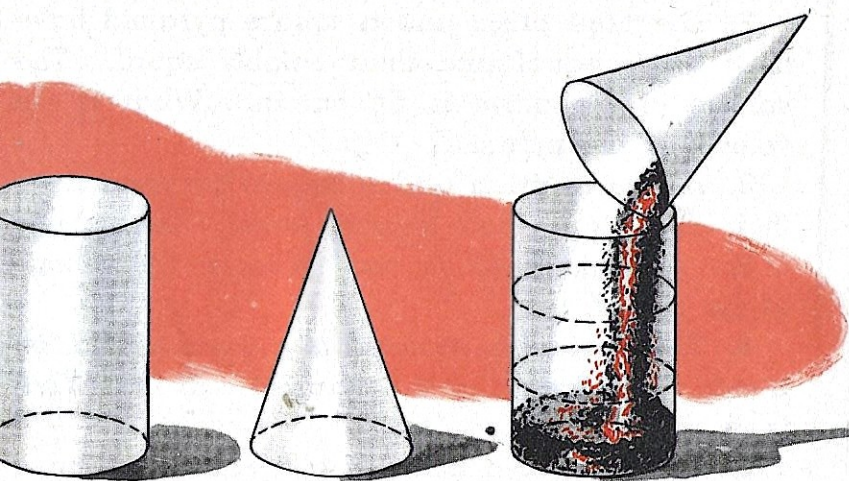
8. $s=10$, $h=9$

11. $s=16$, $h=15$

12. A granite gate post is a prism capped with a pyramid. The prism has a square base 2 ft. on a side and is 8 ft. high. The pyramid has a square base 2 ft. on a side, and is $1\frac{1}{2}$ ft. high. What is the total volume? Make a drawing.

13. The Washington Monument is capped with a solid pyramid of aluminum. The base of this pyramid is a square 5.6 in. on a side, and it is 8.9 in. high. What is its volume?

14. When built the Great Pyramid was 481 ft. high, and each side of its square base was 756 ft. What was its volume?



Volume of a Cone

The bases and the heights of the cylinder and the cone above are equal. Make paper models.

Use sand and show that the volume of the cone is equal to one third the volume of the cylinder.

The volume of the cylinder is given by this formula:

$$V = \pi r^2 h$$

The volume of the cone is given by this formula:

$$V = \frac{\pi r^2 h}{3}$$

1. Find the volume of a cone, the radius of whose base is $3\frac{1}{2}$ in. and whose height is 6 in.

$$V = \frac{1}{3} \times \frac{11}{7} \times \frac{1}{2} \times \frac{7}{2} \times \frac{1}{2} \times \frac{1}{6} = 77$$

The volume of the cone is 77 cu. in.

2. A cylinder and a cone have equal bases and heights. The volume of the cylinder is 78 cu. in. What is the volume of the cone?

3. A cylinder and a cone have equal bases and heights. The volume of the cone is 36 cu. in. What is the volume of the cylinder?

4. Find the volume of a cylinder, the radius of whose base is 7 in. and whose height is 12 in.

5. Find the volume of a cone, the radius of whose base is 7 in. and whose height is 12 in.

Find the volumes of the cones.

6. $r = 3\frac{1}{2}$, $h = 12$

9. $d = 7$, $h = 24$

7. $r = 1\frac{3}{4}$, $h = 16$

10. $r = 3\frac{1}{2}$, $h = 32$

8. $r = 10\frac{1}{2}$, $h = 5$

11. $d = 21$, $h = 10$

12. A conical pile of sand has a diameter of 21 ft. and a height of 12 ft. What is its volume in cubic feet?

13. A paper drinking cup is shaped like a cone. The diameter of the top is $3\frac{1}{2}$ in., and the height is $5\frac{1}{4}$ in. How many cubic inches does it hold?

14. A glass holds about $\frac{1}{2}$ pint. Compare the volume of the paper cup in Problem 13 with the volume of a glass.

15. A paper cone used at soda fountains is $4\frac{1}{2}$ in. deep and $3\frac{1}{2}$ in. across the top. What is its volume in cubic inches?

16. The Indian chief's tent is a cone with a diameter of 14 ft. and a height of 15 ft. How many cubic feet of air are in the tent?



Volume of a Sphere

In the figures above, the diameter and the height of the cylinder and the diameter of the sphere are all equal.

Use sand to show the volume of the hemisphere equals one third the volume of the cylinder.

The volume of the cylinder is given by the formula:

$$V = \pi r^2 h, \text{ or } V = \pi r^2 \times 2r, \text{ or } V = 2\pi r^3.$$

The volume of the hemisphere is given by the formula:

$$V = \frac{2\pi r^3}{3}$$

The volume of a sphere is twice the volume of a hemisphere. The volume of a sphere is given by the formula:

$$V = \frac{4\pi r^3}{3}$$

1. Find the volume of a sphere with radius $10\frac{1}{2}$.

$$V = \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} = 4,851$$

A cylinder and a sphere are related as shown in the figures at the top of page 228.

2. The volume of the cylinder is 63 cu. in. What is the volume of a hemisphere?
3. If the volume of the cylinder is 84 cu. in., what is the volume of the sphere?
4. If the volume of the hemisphere is 36 cu. in., what is the volume of the cylinder? What is the volume of the sphere?
5. The diameter of a cylinder is 14 in. and its height is 14 in. What is its volume?
6. What is the volume of a sphere 14 in. in diameter?

Find the volume of each of these spheres:

7. $r = 7''$
8. $r = 3''$
9. $r = 9''$
10. $r = 1\frac{3}{4}'$
11. $d = 42''$
12. $d = 28''$
13. $d = 70''$
14. $d = 30'$
15. $r = 10''$
16. $d = 35''$
17. $d = \frac{1}{2}''$
18. $r = 4\frac{1}{2}''$
19. A spherical gas tank is 84 ft. in diameter. What is its volume in cubic feet?
20. A spherical balloon is 72 ft. in diameter. How many cubic feet of gas will it hold?
21. A croquet ball is a wooden sphere 4 in. in diameter. What is its volume in cubic inches?
22. A certain gate post is a cylinder with a sphere on its top. The diameter of the cylinder is $17\frac{1}{2}$ in., and it is 72 in. high. The diameter of the sphere is 21 in. What is the total volume of the gate post?

Things You Ought to Know and Remember

I. Only simple cases of the prism, cylinder, pyramid, and cone are studied in this unit. You will find other cases when you study solid geometry in high school.

II. **Lengths** ($h, k, p, c, w, l, s, r, d$) are in units of **linear** measure. Review linear measure table.

Areas (B, L, T) are in units of **square** measure. Review table of square measure.

Volumes (V) are in units of **cubic** measure. Review table of cubic measure.

III. When used together, units of measure must be of like sort. This means that in finding an area or a volume, you may not multiply a number that indicates inches by a number that indicates feet or yards.

IV. Each of the following letters stands for a measure: $L, T, B, V, s, r, d, h, k, p, c, w, l$. Copy these letters in a column, and after each write its meaning.

V. There are fifteen important formulas in this unit concerning lateral area, total area, and volume. Make a list of these formulas, and indicate what each will measure.

VI. Solids will mean more to you if you learn to see them in the things about you. For example, gas and water pipes, rods and wires, and most containers are cylinders. A stick of lumber or a room is a prism. Name others.

The Tip-Off

A prism is 8 inches high, and its base is a square which is 4 inches on a side. Find—

1. p 2. B 3. L 4. T 5. V

A side of the base of a square pyramid is 8 inches, the slant height of the pyramid is 5 inches, and the height is 3 inches. Find—

6. p 7. B 8. L 9. T 10. V

A cylinder is 24 inches high and the diameter of its base is 14 inches. Find—

11. r 12. c 13. L 14. T 15. V

The radius of the base of a cone is 7 inches, and its slant height is 25 inches. If $h=24$, find—

16. d 17. c 18. L 19. T 20. V

The diameter of a sphere is 21 inches. Find—

21. r 22. T 23. V

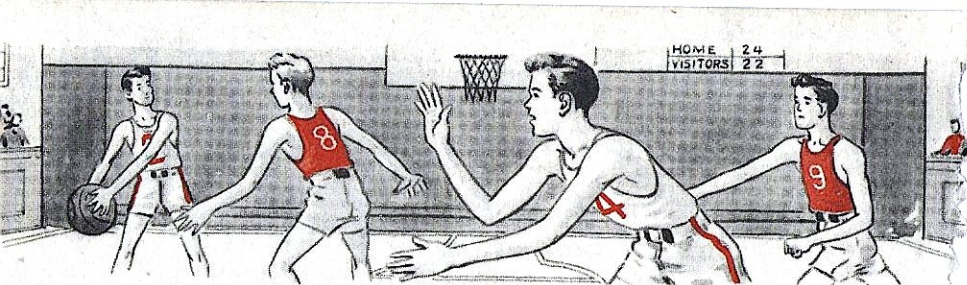
Write the number that belongs in place of each * below.

24. A prism and a pyramid have equal bases and equal heights. The volume of the pyramid is * the volume of the prism.

25. The diameter of the base of a cylinder, the height of the cylinder, and the diameter of a sphere are all equal. The volume of the sphere is * of the volume of the cylinder.

Those who found as many as 24 right answers may work on page 233.





Recovering the Ball

Each letter below stands for a measure used in this unit. Write or tell briefly what each of these measures is.

1. L T B V s r w

2. d h k p c π l

3. Each space in the chart below tells the page where you may find a formula that belongs in that space. Copy the chart on a larger scale, writing in the appropriate formula in place of the page number.

	Prism	Cylinder	Pyramid	Cone	Sphere
L	212	213	214	215	
T	212	213	214	215	216
V	220	221	224	226	228

4. Use your chart and say each of the formulas in words. Instead of $L=ph$, say "The lateral area of a prism is equal to the perimeter of its base multiplied by its height." Work with a partner and do this for each of the formulas.

5. Use each of the formulas in your chart to solve one problem. You may make up the problems or choose them from those given in the book.

Passing the Ball

1. One type of oil barrel is 21 in. in diameter and 28 in. high. What is the volume of this barrel in cubic inches? How many gallons will it hold?

2. A certain tanker carries 3,500,000 gallons of oil as cargo. This is equal to how many 42-gallon barrels of oil?

3. A railroad tank car is a cylinder 77 in. in diameter and 40 ft. long. What is its volume in cubic inches? How many gallons will it hold?

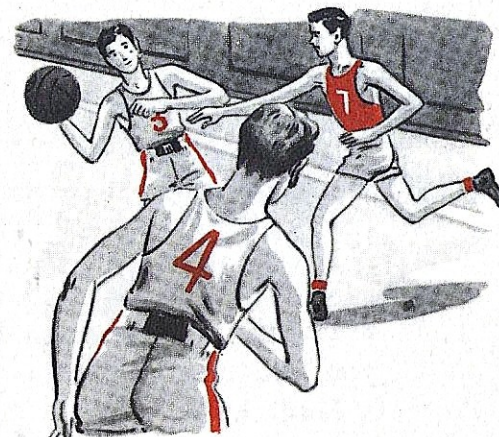
4. An ocean liner uses 510 gal. of oil per hour. How many barrels of oil will it need for a 5-day trip?

5. A pipe line for oil is a cylinder 21 in. in diameter. When full, how many cubic inches of oil are in 88 in. of this pipe line? How many gallons is this?

6. Oil flows in the pipe line (Prob. 5) at the rate of 264 ft. per minute. How many gallons of oil will flow past a given point in a minute?

7. How many gallons of oil will flow past a given point of the pipe line (Prob. 5) in 24 hr.?

8. The pipe line is 300 mi. long. How many hours will it take a particular drop of oil to flow from one end of the line to the other?





Goal for Unit Nine

In solving a problem,
(a) draw a figure showing the parts that are given; (b) select the formula or formulas from below the heavy line that you will need; (c) solve the problem.

Find the—

1. volume of a cylinder when $r=3\frac{1}{2}$, $h=8$.
2. volume of a sphere when $d=7$.
3. total surface of a rectangular solid when $l=8$, $w=5$, and $h=10$.
4. surface of a sphere when $r=10\frac{1}{2}$.
5. total surface of a pyramid whose base is a square 4 in. on a side, and whose slant height is 3 in.
6. volume of a triangular prism when $B=18$, $h=5$.
7. volume of a cone when $r=3\frac{1}{2}$, $h=8$.
8. volume of a square pyramid when a side of the base is 5 and the height is 8.
9. total surface of a cylinder when $r=7$, $h=5$.
10. lateral surface of a triangular prism when $p=23$ and $h=10$.

A. $V = \frac{1}{3} Bh.$

B. $V = \frac{1}{3} \pi r^2 h.$

C. $T = 4\pi r^2.$

D. $L = ph.$

E. $T = s^2 + 2sk.$

F. $T = 2\pi rh + 2\pi r^2.$

G. $T = 2lw + 2lh + 2wh.$

H. $V = \frac{4}{3} \pi r^3.$

J. $V = \pi r^2 h.$

K. $V = Bh.$



Unit Ten

Protecting Investments and Savings

Earlier units have taught you how to save and invest money. It is equally important to know how such savings and investments can be reasonably well protected.

The farm above is Mr. Bell's (p. 151). Will the one who holds a mortgage on this property lose his investment? (p. 152, Ex. 2)

Kinds of Insurance Protection

Mr. Bell believed in protecting his family and his investments. In addition to the insurance protecting the mortgage, he carried theft, fire, hail, flood, and windstorm insurance on his automobile, crops, stock, household furnishings, and clothing. He also carried accident and life insurance on each member of his family.

1. Make a list of any of the insurance protections above on which Mr. Bell is likely to collect for damage done by the storm. See picture, p. 235. Explain why you selected the ones that you listed.

2. How much fire insurance will Mr. Bell receive on the barn and equipment if these are insured for three-fourths their cost (p. 151) and are totally destroyed?

3. What do you think is a reasonable estimate of the damage done to the house and furnishings? To the automobile? Use picture, p. 235, to judge.

4. Was Mr. Bell wise to insure his property?

The four commonest kinds of insurance are fire, automobile, life, and health and accident insurance. Which of these do your parents carry?

5. Bring some advertisements for insurance to school. Pin them up for class study.

6. Find items in a newspaper describing happenings in which some kind of insurance would have been a protection.

Sharing Each Other's Fire Losses

The earliest form of fire insurance was the **mutual association** in which a group of people in a community agreed to share each other's losses from fire. This was fire insurance of a very simple type. No money was paid until a fire occurred. The loss was then divided, each member in the association paying his share.

1. A mutual fire insurance association was formed by 100 farmers. They agreed that in case any one of them suffered loss by fire, each of the 100 members would pay up to \$10 to the member having the loss. What would be the largest fire damages that could be collected for a single fire?

2. Many fires did not cause a full \$1,000 damage. What would each of the 100 members pay if the actual losses from a fire were \$750? \$200? \$500? \$100? \$375?

3. How much would a member of a mutual group of 100 persons pay in a year for fire insurance if there were six fires with the following losses to be shared: \$1000, \$1000, \$500, \$100, \$300, and \$250?



How a Fire Insurance Company Operates

The modern insurance company is a group of stockholders selling insurance protection at a profit. To such a company sharing your fire losses is a business. You pay a small yearly sum, or **premium**, to the company. In return the company issues you an **agreement**, or **policy**, stating that in case your insured property burns, the company will pay up to the amount, or **face**, of the policy to cover your fire losses.

1. Mr. Long built a \$15,000 home. He did not think it could ever be entirely destroyed by fire, and so he insured it for $\frac{3}{4}$ of its value against fire loss. What was the face of his fire insurance policy?

2. Last year Mr. Tamm paid a premium of \$16.50 to insure his \$5,000 house against total loss by fire. The house burned and was completely destroyed. How much insurance did the company pay Mr. Tamm? How could the company afford to pay so much insurance for such a small premium?

3. The owners of 1000 small brick houses together paid \$33,000 in premiums to a certain fire insurance company. During the year 6 of the houses burned and the insurance company paid \$18,000 for losses. How much of the premiums did the company have left for expenses and profits?

4. What was the average premium paid by the owners of the 1000 houses in Problem 3?

5. What was the average insurance paid on the 6 houses that burned?

6. Mr. Jones pays 16¢ for each \$100 of fire insurance that he carries. What is his yearly premium on \$4,000 insurance? On \$6,500? On \$15,000? On \$7,500?

7. What is the premium on a \$7,200 policy for one year at 28¢ per \$100? For 2 years?

The period of time during which the company is liable for loss is called the **term**. As long as the company is bound to pay for loss, the policy is said to be **in force**. When the period is ended, the policy **expires**. The policy can be **renewed** by paying the premium for another term. If the insured fails to pay the premium, the policy is said to **lapse**.

8. Mr. Brown insured his house for \$8,000 on January 2, 1948 for one year. He paid 48¢ per \$100. What was his premium? What was the face of the policy? What was the term of his insurance? When will the policy expire? What will Mr. Brown pay the company at the end of the period if he wishes to renew this same amount of insurance on his house for the same term? What might happen if he forgot to pay his premium and his policy lapsed?

Rates and Risks

A person may carry as much fire insurance as he wishes, provided the insurance does not exceed the value of the insured property. Why is this limit set?

The amount charged for each \$100 of fire insurance is called the insurance rate. Rates vary according to the kind of property, its use, location, nearness of fire fighting equipment, kinds of roofs, and other conditions. Property is called a good or bad **insurance risk** according to the chances that it will be destroyed by fire.

Insurance companies group cities into classes. Cities having the best fire-fighting equipment are charged the lowest rates. Cities with poorer equipment are charged higher rates.

1. Do you think the grouping described above is fair?

2. Name a city that you think should have a low insurance rate. Give your reason for naming this city.

3. Try to bring an expired fire insurance policy to school. Pin it on a bulletin board where the class and you can study it together.

4. Choose the best fire insurance risk from the following. Explain why the others are poorer risks.

A brick house on a farm with no fire protection near.

A single brick dwelling, slate roof, near modern fire fighting equipment.

A public garage for automobile parking.

A chemical plant making explosives.

A brick house in a large city, two families, double garage for cars in the basement of the house.

5. Ask some local fire insurance agent to give you the fire insurance rates charged in your community on the risks listed in the table below. Copy the table on the blackboard putting your local rates in a third column headed C.

6. How do the rates in Column C compare with the rates in Columns A and B? Explain any difference.

Table of Insurance Rates per \$100

"A" means a large city. "B" means a small village.

Type of Property	Annual Rate	
	A	B
Single brick dwelling, approved roof	\$0.14	\$0.48
Single brick dwelling, unapproved roof	.22	.58
Single frame dwelling, approved roof	.30	.76
Single frame dwelling, unapproved roof	.38	.86
Brick apartment house, 20 families	.37	.85
Public garage	3.03	3.48
Fire-proof hotel	2.15	2.95

Fire Insurance Problems

The term of an insurance policy may be one, three, or five years. Periods longer than one year are called **long terms**. Insurance companies give special rates for a long term when the premium is paid in advance.

Long Term Rates

Premium for 3 yr. is $2\frac{1}{2}$ times that for 1 yr.

Premium for 5 yr. is 4 times that for 1 yr.

1. The premium for \$5000 fire insurance for one year is \$15. What will the same policy cost if the term is 3 yr.? 5 yr.?

Use the rates given on page 241. Find the premiums on the following policies.

Type of Property	Size of Town	Face	Term
2. Single brick, approved roof	A	\$4,000	1 yr.
3. Brick apartment, 20 families	B	\$51,000	5 yr.
4. Public garage	A	\$12,000	3 yr.
5. Single frame, unapproved roof	A	\$2,500	5 yr.
6. Single brick, approved roof	B	\$6,000	3 yr.

7. A home owner is about to insure his house for \$8,000 at an annual rate of 72¢ per \$100. How much will he save by paying for 3 years in advance? 5 years?

8. When insured property burns, the insurance

company sends an **adjuster**. What are his duties? How would Mr. Miller's inventory (page 157) be of assistance when an adjuster came if Mr. Miller's store burned?

9. If your family's house furnishings were insured against fire, would you be able to recall all the books, furniture, pictures, silverware, and other things destroyed by a fire? *Each home should have a careful inventory of each insured article, showing the time of purchase, place of purchase, and original cost.* Make an inventory of your own possessions, such as clothes, jewelry, books, games, equipment used in hobbies.

10. Mrs. Wells values her home furnishings and family clothing at \$4,500. What will it cost to insure these at 26¢ per \$100?

11. Mr. Ames had his home insured for \$6,000. After a fire, the adjuster estimated that the house could be put in its original condition for \$1,800. How much will Mr. Ames be paid by the insurance company?

12. Explain the effect of the following on insurance rates:

Exchanging outdated fire-fighting trucks for the very latest equipment.

Leaving a building unoccupied most of the time.

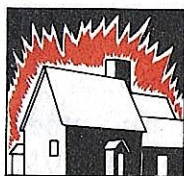
Hiring a night watchman for a factory.

Replacing a wood shingle roof with a slate roof.

Annual Fire Losses in the United States

FIRES COST U. S. OVER A MILLION DOLLARS A DAY

660,000 Fires Annually Attack



340,000 Homes



29,000 Factories



12,000 Public Buildings,
Schools, Hospitals, Etc.



279,000 Miscellaneous
Buildings

LEADING FIRE CAUSES EACH YEAR



Matches and Smoking 130,000 Fires



Electrical
74,000 Fires



Sparks on Roofs
55,000 Fires



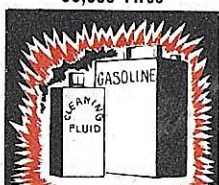
Defective Chimneys
53,000 Fires



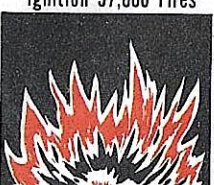
Rubbish and Spontaneous
Ignition 37,000 Fires



Defective Heating Equipment
45,000 Fires



Flammable Liquid
25,000 Fires



All Other Causes
207,000 Fires

The Battle with Fire

1. Fires destroy necessary materials, leaving useless ashes in their place. Would you say fires are a loss to the community as well as to the property owner? Explain.

2. Do you think fire insurance ever fully covers the property owner's losses when you consider time, work, and worry? Give a good illustration.

Page 244 will supply missing facts for the following.

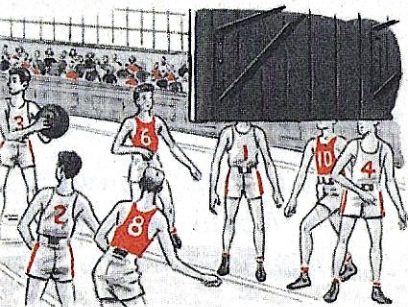
3. Estimate the yearly cost of fires in the United States.

4. What per cent of the total number of fires in the United States are homes? Factories? Public buildings? Miscellaneous buildings? (Carry only to second decimal place.)

5. Use the per cents found in Problem 4. Make a circular graph showing what part of the 660,000 annual fires is in each of the following: homes, factories, public and miscellaneous buildings.

6. Make a bar graph showing the number of fires each year from each of the sources shown under leading fire causes. Let $\frac{1}{16}$ " represent 5,000 fires.

7. Make a set of safety rules including the following fire hazards: electrical equipment, matches, rubbish, chimneys, roofs, gasoline, cigarettes, ashes, oil mops, dust from carpet sweepers, and fly sprays containing kerosene.



Free Throws

You will use many decimals in this unit. Choose a partner and do this decimal practice. Check after each throw. If either of you needs extra practice, do page 334.

First Throw: Use a short method and multiply each of these numbers by 10; by 100; by 1000.

1. $5.6 \quad .39 \quad .045 \quad .0255 \quad 42 \quad 50.57$

2. Multiply each of the numbers above by 60; 30; 2000.

Second Throw: Find the products.

3. $5.4 \quad 3.25 \quad 9.887 \quad .1952 \quad 91.025 \quad .79$
 $\underline{.08} \quad \underline{.07} \quad \underline{49} \quad \underline{107} \quad \underline{.5} \quad \underline{50}$

4. $9.2 \quad 6.39 \quad 9.904 \quad .8729 \quad 72.004 \quad .85$
 $\underline{5.5} \quad \underline{.61} \quad \underline{.045} \quad \underline{.12} \quad \underline{2000} \quad \underline{.46}$

Third Throw: Use a short method and divide each of these numbers by 10; 100; 1000.

5. $125 \quad 5684 \quad 6.5 \quad 79.352 \quad 25000 \quad 80$

6. Divide each of the numbers in row 5 by 20; by 300; by 6000.

Fourth Throw: Find the quotients.

7. $3.2 \overline{)6.4} \quad .188 \overline{)79.36} \quad 4.7 \overline{)25.472}$

8. $.28 \overline{)8.4} \quad .0026 \overline{)2.4232} \quad .39 \overline{)276.12}$

9. $2.4 \overline{)7.2} \quad .075 \overline{)2.1225} \quad .0092 \overline{)583.28}$



Automobile Insurance

In the United States automobile accidents take the lives of more than 100 persons daily. Thousands are injured or permanently crippled each year. Fire, theft, and damage from storms, collisions, and other accidents cause losses amounting to millions of dollars annually. Car owners may be held responsible for damage done by their cars to the life, health, or property of others. In order to protect owners from loss and costly payments for damages, insurance companies offer several types of insurance.

Kinds of Automobile Insurance

A car owner may carry any of the following insurance protections.

Property damage covering damage done to other people's cars or property.

Public liability covering loss from injuries or death caused by insured car.

Collision, or upset, protecting insured car against loss caused by collision with another car or object.

Fire and lightning covering damage to insured car.

Theft of car and pilferage covering loss of tools, spotlights, etc. from insured car.

Comprehensive covering breakage of glass and loss to insured car caused by fire, theft, explosion, earthquake, windstorm, hail, water, flood, and moving objects, exclusive of collision.

1. If you carried comprehensive coverage would you also carry fire and theft? Explain.

2. If glass in your car was broken in an accident with another car, would your comprehensive insurance cover your loss? Why?

3. Mr. Wells had three types of automobile insurance: comprehensive, property damage, and public liability. He struck another car and the other man's repair bill was \$275. Which one of his policies paid for this damage?

4. Mr. Wells' car was damaged to the extent of \$180. Did he have to pay this himself? What type of insurance covers this loss?

5. Three passengers in the car struck by Mr. Wells were seriously injured. They sued him and the court awarded them a total of \$12,000 damages. Which policy protected Mr. Wells against such loss?

Public, or bodily injury liability, is usually covered to a certain limit, as \$5000 for each person and \$10,000 limit for each occurrence, or \$20,000 for each person and \$40,000 for each occurrence.

6. Mr. Wells carried \$5000/\$10,000 limit in his public liability policy. Did his insurance cover the damages awarded in Problem 5?

7. Public liability insurance, limits \$5000/\$10,000, costs \$15.30 a year on a certain car. For an additional premium of \$5.51 a policyholder can get \$25,000/\$50,000 protection. Is it wise to have the insurance with the higher limits? Why? What will the 25/50 (read 25, 50) protection cost?

8. Theft insurance on a low-price car in a small town costs \$.80 per \$100 of value. For the same car it costs \$3.40 per \$100 in a large city. Why does theft insurance cost less in a small town?

9. Why does theft insurance cost more on a low-priced car than on a higher priced car?

Automobile Insurance Rates

Rates for automobile insurance are so many and varied that a complete list would fill a large book. The cost varies according to the state one lives in, the make and model of the car, its age, the purpose for which it is used, and the amount of insurance.

1. Mr. Smith owns a new, higher-priced car. Find the total cost of his insurance for one year at the following rates.

Comprehensive—fire, theft, etc.	\$31.10
Property damage—up to \$5000	8.00
Public liability—\$5000/\$10,000	34.00
Collision (\$50 deductible)	<u>26.00</u>
	\$.

The term "\$50 deductible" means the company will pay damages in excess of \$50. It is left for the owner to pay any damages of \$50 or less. If the damage is \$38, the owner pays all; if the damage is \$87, the company pays \$37 and the owner pays \$50.

Below is a short table showing the cost in a certain state of four kinds of insurance on three lower-priced cars when new.

Name of Car	Comprehensive	Property Damage	Public Liability 5/10	Collision \$50 Deductible
A.	\$18.50	\$7.00	\$33.00	\$20.00
B.	\$15.95	\$7.00	\$33.00	\$24.00
C.	\$16.50	\$8.00	\$34.00	\$28.00

2. Find the total cost for all four kinds of insurance on each car listed above.

3. Mr. Green had an accident killing two people. He was sued, and the court awarded the families of the victims \$5000 for each death. Mr. Green carried no automobile insurance. He had to sell his \$6,500 home, take \$1,820 savings from the bank, and pay the remaining amount from his earnings, month by month. If he paid \$35 each month, how long did it take to pay the entire amount?

4. Mr. Green was trying to save money by not carrying insurance. Was he wise? What would public liability insurance have cost if he was driving a new car of the make listed as A in the table on page 250?

5. How much would Mr. Green have saved if he had been covered by public liability insurance at the time of the accident?

6. Mrs. Poe parked her car and went shopping. Another car crashed into it. She carried \$50 deductible collision insurance. What did the company pay if the repairs were \$95? What would the company pay if she carried \$25 deductible?

7. Why do people carry deductible collision insurance?

8. If the cars pictured on page 247 had full coverage, on which policies will the owners collect?

9. If you were going to carry only one kind of automobile insurance, which would you choose?

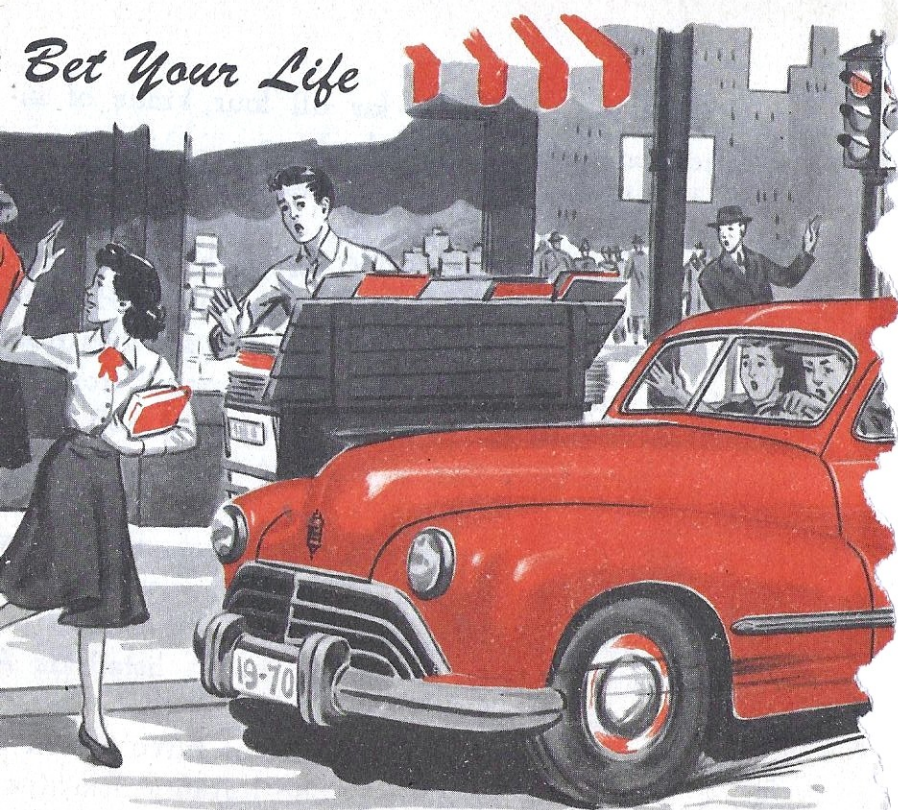


Table 1. Types of Accidents Resulting in Deaths and Injuries

	Persons Killed	Per Cent	Persons Injured	Per Cent
Collision with:				
Pedestrian	10,070	29.7	171,910	13.2
Automobile	10,950	32.3	781,670	60.0
Horse-drawn vehicle	110	.3	3,750	.3
Railroad train	2,110	6.2	7,500	.6
Street car	60	.2	16,990	1.3
Other vehicle	240	.7	6,040	.5
Fixed object	2,980	8.8	107,390	8.2
Bicycle	570	1.7	21,600	1.7
Non-collision	6,490	19.2	173,900	13.3
Miscellaneous	320	.9	11,250	.9
TOTAL	33,900	100.0	1,302,000	100.0

Don't Take Chances

When you take chances in traffic, you are betting your life! Are you one of the 1,335,900 persons likely to be injured or killed in traffic this year?

Use the table on page 252 to get facts needed in the following problems.

1. How is the figure 1,335,900 found?
2. Write 1,335,900 rounded to the nearest ten thousand. Find what per cent the round number is of the population of the United States rounded to the nearest million.
3. Use the per cent found in Problem 2 and find how many persons in a hundred were injured or killed in the United States during the year covered by the table, page 252. How many per 1000?
4. Which type of accident caused most injuries? Most deaths? How can you tell?
5. Are bicycles more dangerous in traffic than horse-drawn vehicles? Show how you found your answer.

6. According to the table are you safer walking than riding in an automobile? Explain.

7. According to facts given in the table are accidents involving a train more likely to be fatal than a streetcar accident?

8. The number of persons killed in streetcar accidents is what per cent of the total streetcar accidents?

Bet Your Life

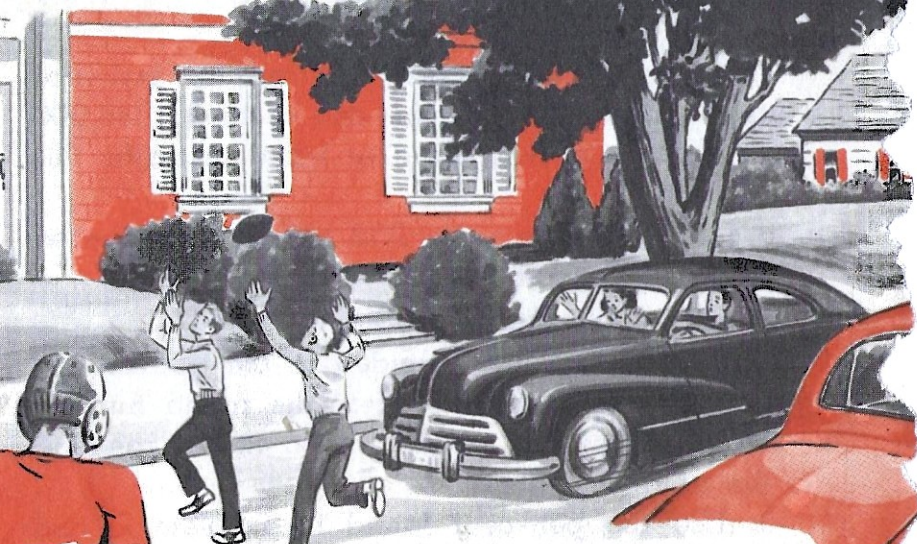


Table 2. Actions of Pedestrians Resulting in Deaths and Injuries

	Pedestrians Killed	Per Cent	Pedestrians Injured	Per Cent
Crossing at intersections:				
With signal	290	2.9	10,320	6.0
Against signal	780	7.7	20,970	12.2
No signal	1,190	11.8	19,770	11.5
Diagonally	180	1.8	2,920	1.7
Crossing between intersections	2,770	27.5	46,770	27.2
Waiting for or getting on or off street car	40	.4	1,020	.6
Standing on safety isle	20	.2	340	.2
Getting on or off other vehicle	110	1.1	2,410	1.4
Children playing in street	870	8.6	21,660	12.6
At work in road	240	2.4	3,950	2.3
Riding or hitching on vehicle	140	1.4	2,240	1.3
Coming from behind parked car	840	8.3	20,970	12.2
Walking on rural highway	1,960	19.5	7,740	4.5
Not on roadway	360	3.6	8,250	4.8
Miscellaneous	280	2.8	2,580	1.5
TOTAL	10,070	100.0	171,910	100.0

Learning from Tables

Pain and death walk the streets and highways hand-in-hand with carelessness. Are you a careless pedestrian? Use facts from the table on page 254 to learn what carelessness cost pedestrians.

1. Which act of pedestrians resulted in the greatest number of deaths in the year summarized in the table? In the greatest number of persons injured? Find the total for both.

2. Find the total number of persons killed or injured at street intersections.

3. What per cent of the injuries to persons occurred at intersections? What per cent of the deaths?

4. Which is larger, the percentage of deaths from accidents at intersections or between intersections? Is there a great difference? Is the same true of accidents resulting in injuries?

5. What per cent of the accidents to children playing in the street resulted in death?

6. How many persons were injured or killed as they walked along rural highways? How does this rank with other accidents reported in the table? On which side of the highway should a pedestrian walk?

7. Does a very large part of accidents to pedestrians result from coming from behind parked cars? How can you estimate this?

Gaining Strength in Problem Solving

The approximate cost of carrying air express is given by the formula,

$$C = \frac{W \times D}{25}$$

In this formula, C is the cost in cents, W is the weight in pounds, and D is the distance rounded to the nearest hundred miles.

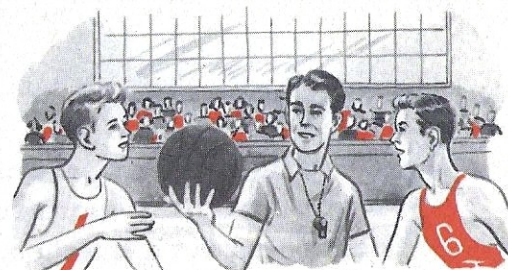
Find the cost of air express of each of these shipments.

1. Machine part, weighing $2\frac{1}{4}$ pounds, from New York to Kansas City, air distance 1,104 miles.
2. Orchids, weighing $1\frac{3}{4}$ pounds, from New Orleans to Chicago, air distance 857 miles.
3. Valuable papers, weighing $3\frac{1}{2}$ pounds, from Washington to Norfolk, air distance 143 miles.
4. Medicine, weighing $\frac{3}{4}$ pound, from New York to Boston, air distance 184 miles.
5. Nylon hose, weighing $4\frac{1}{2}$ pounds, from New York to Chicago, air distance 724 miles.
6. Party dress, weighing $3\frac{3}{4}$ pounds, from Dallas to Houston, air distance 241 miles.
7. Vaccination serum, weighing 5 pounds, from Chicago to Kansas City, air distance 405 miles.
8. Jewelry, weighing $2\frac{1}{2}$ pounds, from Boston to New York, air distance 184 miles.



The Tip-Off

Find the annual premium for fire insurance on—



	Property Value	Part of Value Insured	Rate per \$100
1.	\$15,000	$\frac{2}{3}$	47¢
2.	\$7,000	$\frac{3}{4}$	84¢
3.	\$12,000	$\frac{7}{8}$	56¢
4.	\$5,000	$\frac{7}{8}$	32¢

5. Mr. Hill insured his home for \$8,500 and the furnishings for \$4,500. The rate for 3 years is 60¢ per \$100 on the house and 75¢ per \$100 on the furnishings. What premium did he pay to insure both for 3 years?

6. Mr. Hollis insured his warehouse for \$120,000 for a period of 5 years. The premium for 5 years is 4 times the premium for 1 year. What did he save by taking the insurance for 5 years if the annual rate is \$1.05 per \$100 valuation?

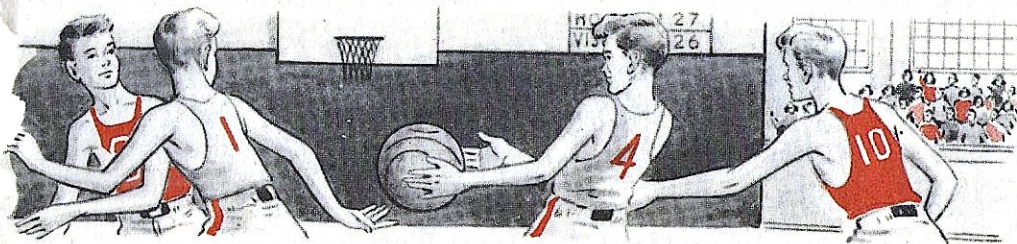
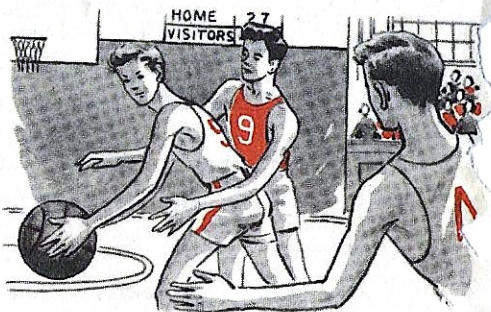
7. Mr. Lot carried comprehensive, \$50 deductible collision, and \$25/50 public liability insurance on his car. He had an accident damaging a car to the extent of \$286, and his own, \$135. How much of the repair bills did he pay? What did the insurance company pay?

If you did not work 6 of the problems above correctly, take the insurance quiz on page 258.

Recovering the Ball

Try this insurance quiz. If you and your partner cannot answer a question, study the page given in parentheses after it.

1. List kinds of automobile insurance. (248)
2. Which kind of insurance pays for injury to a person or loss of life? (248)
3. What is meant by the premium paid on insurance? (238)
4. What is an insurance policy? (238)
5. What is meant by the term of the insurance? (239)
6. What are the long-term rates on fire insurance? (242)
7. Find the premium for 3 years on a \$500,000 fire-proof hotel located in a large city. Get the yearly rate from the table (241) and the long-term rate from p. 242.
8. What is comprehensive automobile insurance? (248)
9. Explain \$10,000/\$20,000 public liability insurance. (249)
10. A man carrying \$5,000/\$10,000 public liability was sued by the families of three persons who were killed in an accident involving his car. The families were awarded \$5,000 for each victim. Did his insurance cover this amount? (249)



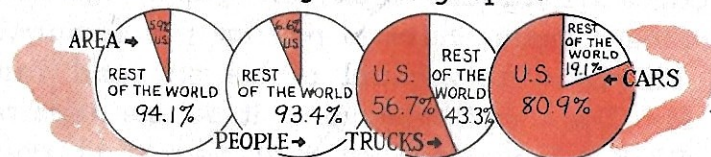
Passing the Ball

1. Make up five insurance problems like Problems 2-7, page 242. Use the facts given in the table on page 241. Solve your problems.

2. Rewrite the seven problems on page 257, substituting other reasonable amounts in place of the numbers used in the problems. Solve the new problems.

3. Take a group of four or five pupils now working page 258, and have them solve the problems you wrote above.

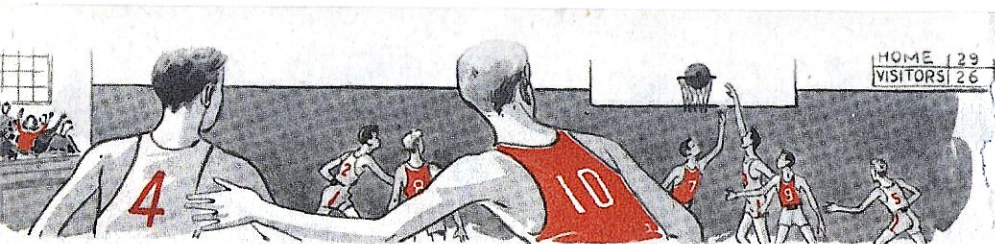
Study the following circle graphs.



4. What per cent of the world's supply of trucks is registered outside the United States?

5. There were 28,053,693 registered passenger cars in the United States when the graphs above were made. How many passenger cars were registered in the world at that time?

6. How can you tell from the graphs above that the United States has a very difficult traffic problem?



Goal for Unit Ten

1. Mr. Rogers owns a \$9,000 house. He insured it at 80% of its value at 65¢ per \$100. What is his annual premium?

2. What is the cost of insuring a house valued at \$6,000 for $\frac{3}{4}$ of its value at 75¢ per \$100?

3. A home owner is about to insure his home for \$4,000 at an annual rate of \$1.10 per \$100. How much will he save by taking 5 years' insurance at 4 times the annual rate?

4. Sam's father carried comprehensive automobile insurance. One day Sam shut the door of the car too hard and broke the glass. Did the insurance company have to pay for this damage?

5. Mr. Estes lost control of his car and ran into Mr. Doe's car, damaging it so the repairs cost \$213. His insurance that year had cost him \$63.75. What did he save by carrying insurance?

6. An apartment house valued at \$140,000 is insured for 90% of its value. What is the premium for 3 years if the annual rate is 55¢ per \$100?

7. Mr. Hale pays \$11.70 for comprehensive insurance, \$9.00 for property damage, \$32.50 for public liability. Find his total premium.



Life Insurance

Life insurance is the largest business in the world. It is so common that there is scarcely a boy or girl of your age in the United States today who does not know something about life insurance. One large insurance company alone has over 32 million policy holders, carrying a total of more than 43 billion dollars' worth of life insurance. It takes ten miles of files to keep the records of its customers. Every minute of the day this company pays an average of \$5,772 to policy holders, and every day it adds about 3,200 new policies to its files. The picture above shows policies of an average day being prepared for mailing.

The Language of Life Insurance

1. Inquire if anyone at your home carries life insurance. If so, ask to have the policy explained to you.

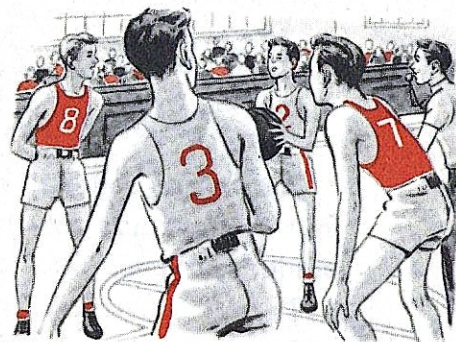
2. Get specimen policies from local life insurance agents. Ask them to explain the policies. Post the policies in the classroom for group study.

3. Arrange an interview with a local life insurance agent. Ask him to explain the different kinds of life insurance, rates, dividends, and tables of mortality. Take careful notes on what he says. You can be a great help to the class during the work of the following pages on life insurance.

You will find in these interviews that the language of life insurance is very much the same as in fire insurance. Persons carrying life insurance receive an **agreement**, or **policy**, from the company. The policy states the **benefits** to the insured and the **conditions** under which the policy is **in force**. A life insurance policy names the **beneficiary**, or the one to whom the face of the policy is to be paid in the event of the death of the insured. Premiums are paid annually, and failure to pay premiums within 30 days of date when due will cause the policy to lapse. Under certain conditions a lapsed policy may be **reinstated**. Many times a health examination is one of the conditions.

Free Throws

Practice the following to improve your skill with fractions and mixed numbers. Work with a partner. Check after each throw.



First Throw: Find the sum of each group.

- | | | |
|---|--|--|
| 1. $\frac{3}{8}, \frac{2}{3}, \frac{3}{4}$ | $\frac{5}{12}, \frac{7}{8}, \frac{1}{4}$ | $\frac{7}{12}, \frac{3}{8}, \frac{2}{3}$ |
| 2. $\frac{5}{12}, \frac{3}{4}, \frac{4}{5}$ | $\frac{7}{8}, \frac{5}{6}, \frac{3}{5}$ | $\frac{9}{16}, \frac{3}{4}, \frac{2}{3}$ |
| 3. $7\frac{11}{16}, 3\frac{7}{8}$ | $2\frac{4}{5}, 14\frac{3}{4}$ | $5\frac{5}{9}, 12\frac{3}{4}$ |
| 4. $4\frac{5}{6}, 9\frac{11}{12}$ | $24, 18\frac{4}{5}$ | $12\frac{5}{8}, 15$ |

Second Throw: Subtract.

- | | |
|---|--|
| 5. $8\frac{1}{3}$ from $12\frac{1}{2}$ | $124\frac{1}{2}$ from $302\frac{1}{4}$ |
| 6. $3\frac{3}{4}$ from $7\frac{1}{6}$ | $840\frac{5}{6}$ from $923\frac{5}{8}$ |
| 7. $6\frac{1}{2}$ from $16\frac{2}{3}$ | $7\frac{1}{2}$ from $8\frac{1}{3}$ |
| 8. $221\frac{2}{3}$ from $418\frac{1}{2}$ | $838\frac{7}{8}$ from $933\frac{1}{3}$ |
| 9. $2\frac{2}{3}$ from $18\frac{1}{6}$ | $7\frac{5}{6}$ from $8\frac{1}{3}$ |
| 10. $16\frac{1}{2}$ from 25 | 12 from $45\frac{5}{6}$ |

Third Throw: Multiply.

- | | | | |
|-------------------------------------|--------------------------------|----------------------------------|-----------------------------------|
| 11. $\frac{3}{4}$ of 8 | $\frac{5}{6}$ of 24 | $18 \times \frac{2}{3}$ | $\frac{3}{8} \times 32$ |
| 12. $\frac{5}{6}$ of $\frac{1}{2}$ | $\frac{2}{3}$ of 16 | $\frac{2}{3} \times \frac{5}{8}$ | $5\frac{1}{2} \times \frac{1}{4}$ |
| 13. $\frac{3}{4}$ of $1\frac{1}{4}$ | $\frac{1}{5}$ of $\frac{5}{6}$ | $\frac{7}{8} \times \frac{8}{9}$ | $5\frac{3}{8} \times \frac{1}{2}$ |
| 14. $\frac{1}{3}$ of $\frac{3}{4}$ | $\frac{5}{8}$ of 32 | $6\frac{2}{3} \times 3$ | $\frac{3}{4} \times \frac{5}{9}$ |

Fourth Throw: Divide.

- | | | | |
|--------------------------------------|---------------------------------|----------------------------------|----------------------------------|
| 15. $\frac{1}{4} \div \frac{1}{2}$ | $6\frac{1}{2} \div \frac{1}{3}$ | $2\frac{1}{2} \div 2$ | $\frac{9}{16} \div \frac{5}{8}$ |
| 16. $6\frac{3}{4} \div 3$ | $\frac{4}{5} \div 1\frac{1}{2}$ | $\frac{7}{12} \div \frac{5}{6}$ | $\frac{7}{8} \div 1\frac{2}{5}$ |
| 17. $\frac{5}{8} \div \frac{5}{6}$ | $3\frac{3}{7} \div \frac{8}{9}$ | $2\frac{2}{3} \div \frac{4}{5}$ | $5\frac{1}{4} \div \frac{7}{8}$ |
| 18. $3\frac{1}{5} \div \frac{8}{15}$ | $\frac{7}{8} \div 5\frac{1}{4}$ | $2\frac{1}{4} \div 1\frac{1}{2}$ | $\frac{5}{12} \div 3\frac{3}{4}$ |

Kinds of Life Insurance

There are several kinds of life insurance. The four main kinds are term insurance, ordinary life, limited payment life, and endowment.

Term insurance is the simplest and cheapest. A term policy is written for a set number of years, as 1 year, 5 years, 10 years. If the insured person dies within the term stated in the policy, the insurance company pays the beneficiary the face of the policy. If the insured person is living at the end of the term, the company keeps the money paid for premiums, and nothing is paid to the insured or his beneficiary.

The table below shows sample rates for term insurance for a \$1000 policy.

1. Mr. Jones took out a \$1000 term insurance policy to run 10 years. He is 30 years old. What premium will he pay each year for the next 5 years?

Term Insurance Rates 10-Yr. Term	
Age	Premium
25	\$11.60
26	11.69
27	11.78
28	11.89
29	12.01
30	12.14

2. If he died 3 years after the policy was written, what did his beneficiary receive?

3. If he did not die within 10 years, what did the protection cost him?

4. What would his policy have cost if he had been 26 years old at the time it was written?

5. What would he pay annually for a \$5000 policy at age 28? Age 25?

A person taking out ordinary life insurance is required to pay a premium each year as long as he lives. When he dies, the face of the policy is paid to his beneficiary. Ordinary life insurance is more expensive than term insurance because at some future date the company must pay the face of the policy if the policy is kept in force.

In limited payment life insurance the premiums are paid in a set number of years, as 5, 10, or 25 years, but the policy continues in force until the insured person dies. The face of the policy is then paid to the beneficiary. If a person thinks his earnings are likely to be very small in later years, this is a good kind of insurance to carry. The premiums can be paid during the early earning years, and the protection will continue through the life of the insured.

Some persons like to plan so their life insurance is a protection plus a savings plan. Such persons take out endowment insurance. *The endowment policy is a special form of the limited payment. The insured decides how many payments he wishes to make, as 10, 15, or 20. If he dies before the last premium is paid, his beneficiary is paid the face of the policy. If he is alive when all payments are made, he gets the amount of the policy.*

Finding the Premium

The premium paid for life insurance depends on the age of the person and the kind of insurance. The table below gives the rates charged by one company for four types of insurance.

Annual Premium Rates Per \$1000 Insurance

Age	Ordinary Life	20-Payment Life	20-Year Endowment	10-Year Term
21	\$18.72	\$24.46	\$44.10	\$8.13
22	19.04	24.84	44.19	8.19
23	19.38	25.27	44.26	8.23
24	19.74	25.72	44.37	8.28
25	20.11	26.18	44.45	8.32
26	20.53	26.65	44.56	8.38
27	20.97	27.18	44.68	8.44
28	21.31	27.71	44.79	8.50
29	21.86	28.05	44.92	8.59
30	22.31	28.41	45.11	8.71

1. Which is the most expensive of the four types of insurance shown above? The cheapest? Explain this difference in price.

2. How much more will a person 30 years old pay for a thousand-dollar ordinary life policy than if he had taken this insurance at 21 years of age?

Find the yearly premium on these policies:

3. 20-yr. endowment for \$1000, age 26.

4. Ordinary life for \$5000, age 22.

5. 20-payment life for \$2000, age 29.

6. Make up 5 problems like problems 3-5 using the facts given in the table. Have other pupils work them for you to check.

7. Three men, A, B, and C, each 25 years of age, bought a \$1000 life insurance policy. Mr. A bought ordinary life, Mr. B bought 20-payment life, and Mr. C bought a 20-year endowment. What annual premium did each pay?

8. At the end of 20 years, how much will A, B, and C have paid, not counting any earnings that the company might pay?

9. At the end of 20 years, what does each have to show for what he has paid?

10. Which policy do you consider best for a man 25 years of age with low earnings and a family to protect?

11. Which type ought a man 25 years of age choose if he has good earnings and wishes to protect his family until the children are grown, but also wants to have an investment beyond this protection?

12. Mr. Elkins was just starting business. He had to go into debt to buy his stock. To protect his family and creditors until his business was paying, he took out \$15,000 worth of 10-year term insurance. What was his yearly premium if he was 30 years old?

13. If Mr. Elkins died in 8 years, what would he leave to his family and creditors?

14. If he lived until the policy expires, what would he have paid to protect his family and creditors?

Insurance Dividends

Many insurance companies share their earnings with the policyholders. The earnings paid to policyholders are called **dividends**. When a policyholder receives a dividend check he may cash it at the company office, apply it on his next premium, or leave the earnings with the company at interest and collect the amount later.

The following table shows that the net cost of insurance, when dividends are considered, is considerably less than the premiums (p. 264).

10-Year Term Insurance

Age	Premium	Average Dividends	Net Cost
25	\$11.60	\$3.48	\$8.12
26	11.69	3.49	8.20
27	11.78	3.50	8.28
28	11.89	3.51	8.38
29	12.01	3.54	8.47
30	12.14	3.54	8.60

1. Mr. Kent, who took out his policy when he was 29 years old, was about to pay his premium when a dividend check came through the mail. If the check was for \$3.54 and he applied it on his premium, how much did he send to pay his next premium?

2. Do you think dividends would be the same amount each year? For all companies? For all kinds of policies?

Interesting Insurance Information

Insurance companies make these four divisions of the money they receive: money for **operating expenses**; money to **pay policies** that become collectible; a **reserve**, or money put aside to meet any emergency; money for **investment** in good stocks, bonds, mortgages, or loans to policyholders.

Most policies have a **loan value**. That is, the policyholder needing money can borrow a limited amount on his policy, paying interest to the company.

Many policies have a **cash surrender value**. That means a policyholder who does not wish to carry his insurance any longer can get back a part of the premiums paid by surrendering his policy to the company.

Certain policies provide a choice of **paid-up insurance** or **extended insurance** to policyholders not wishing to pay further premiums. Paid-up insurance means the policy will continue in force until the death of the insured, but the beneficiary will receive only a part of the face of the policy. Extended insurance allows the full protection of the policy for a few years more. If the insured lives longer than the extended time, the protection stops, and no further benefits are in force. When a policy grants special benefits like loans, cash value, paid-up or extended insurance, there is a table of such values in the policy.

Table Showing Special Values of a Policy

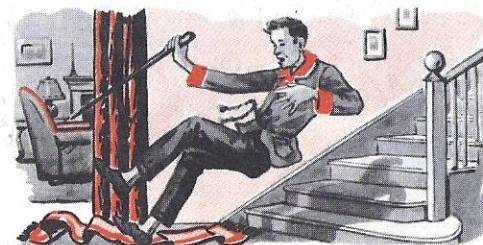
Age 30 Ordinary life: \$1000 Premium \$22.31					
Years in Force	Loan Value	Cash Surrender Value	Paid-up Insurance	Extended Insurance	
				Yr.	Days
1	\$ 00.00	\$ 00.00	\$ 00.00	0	0
2	00.00	00.00	00.00	0	0
3	16.00	18.00	45.00	2	78
4	28.00	31.00	77.00	3	311
5	42.00	45.00	109.00	5	193
10	112.00	120.00	261.00	13	114

Use the facts given in the table and answer the following:

1. How much was paid in premiums in 3 years? How much would be lost by surrendering it for cash at the end of 3 years?
2. Why is the loan value always less than the cash value?
3. After 10 years the holder of the policy above could not pay his premiums. He took paid-up insurance and stopped paying premiums. What will his beneficiary get when he dies?
4. If the person mentioned in problem 3 had taken extended insurance and had died 10 years later, what would his beneficiary get?
5. What could he borrow on the policy at the end of 4 years if the face was \$5000? What would be the paid-up value? Cash value?

Health and Accident Insurance

Many people need all they earn to support their families. If they were sick or injured, there would be no money to pay living and medical expenses. **Health and accident insurance** offer protection against loss of income and provide money for medical care. They may be bought separately or together.

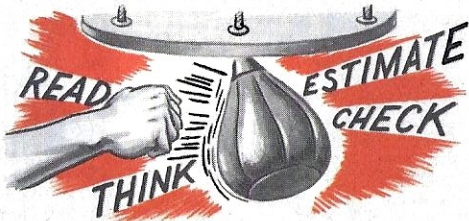


1. Mr. Robb carried a health policy that paid him \$35 a week when he was sick, not exceeding 52 consecutive weeks. What does he get from the insurance company if he is sick 20 weeks? 52 weeks? 60 weeks?

2. Miss Smith has a health policy that pays \$25 a week for illness up to 52 weeks and an additional 50% up to 8 weeks if the illness necessitates her being in a hospital. What will the company pay her if she is sick in the hospital 7 weeks? In the hospital 2 weeks and at home 9 weeks?

3. Mr. Ems has an accident policy which pays hospital and doctor bills up to \$500 for any single accident. He had an accident and his hospital bill was \$324.75 and his doctor bill was \$250. How much did his insurance pay? What did he pay?

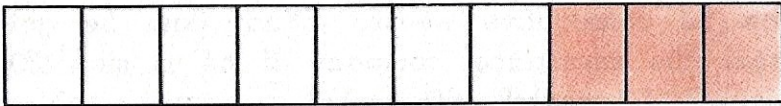
Gaining Strength in Problem Solving



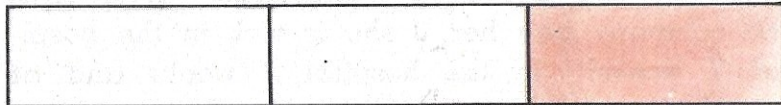
Study the problems and diagrams below. Copy the diagrams. Show with labels what the shaded area of each

represents, what the unshaded area represents, what each space represents, and what the whole area represents. Solve the problems.

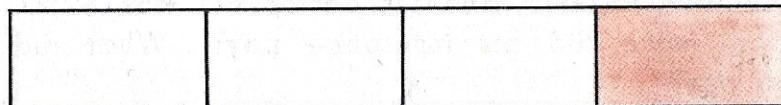
1. Jack paid \$28 for a bicycle. This was at a discount of 30% of the marked price. What per cent of the marked price was the selling price? What was the marked price?



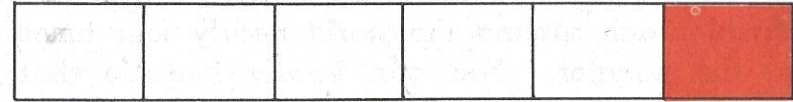
2. Mr. Brown bought a horse for \$180 and sold it at a loss of $33\frac{1}{3}\%$ of the cost. What per cent of the cost was the selling price? What was the selling price?



3. A merchant sold a suit for \$50 which gave him a profit of 25% of the selling price. What per cent of the selling price was the cost? What was the cost?



4. Sam sold his skates for \$6, gaining 20% on what they cost him. Find the cost.



Draw a diagram and solve each of the following problems. Check each answer.

5. Ted paid \$32 for a suit. This was at a discount of 20% of the marked price. What was the marked price?

6. Bill sold his skates for \$4, losing 20% of what he paid for them. How much did he pay for them?

7. Mr. Hill sold a horse for \$180, gaining 25% on what he paid for it. How much did he pay for the horse?

8. The sale price of a hat was \$8. This was $33\frac{1}{3}\%$ less than the regular price. What was the regular price?

9. A grocer paid \$1.10 a bushel for apples and sold them at \$1.65 a bushel. What was his per cent of mark-up based on the cost? What was the per cent of mark-up based on the selling price? You will need to make two diagrams for this one.

10. Mr. Swift has gone 70 miles of a trip of 105 miles. What per cent of the trip has he gone?

11. Bob rode his bicycle 8 miles and then walked 25% as far as he had ridden. How far did he walk?

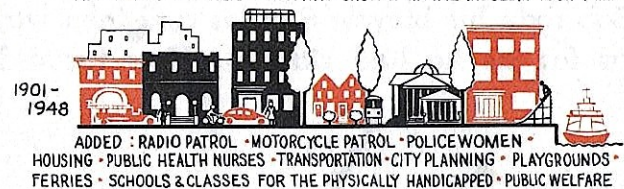
Expenses of Government

One billion one-dollar bills placed end to end would reach around the world nearly four times at the equator. You can hardly imagine that much money, and yet our different governments spend many times that amount in a single year.

1. Make a list of benefits that you are receiving from money spent by the government.

2. The pictures below show that services performed by government have increased from year to year. Explain the pictures. How does this affect the amount of taxes citizens must pay?

CITY FUNCTIONS



Sharing the Cost of Government

The government, like any other great industry, has an income, borrows money, owns property, hires workers, has overhead expenses, and transacts business. Each citizen is a stockholder, and the benefits he receives from the government are his dividends for paying taxes or sharing the expenses of government.

1. Make a list of services which your government performs for you and your family. Which would you miss most if you did not have it?

2. Explain why a good tax system should—

Raise enough money to operate the government and meet changing needs

Be simple enough to be understood by all

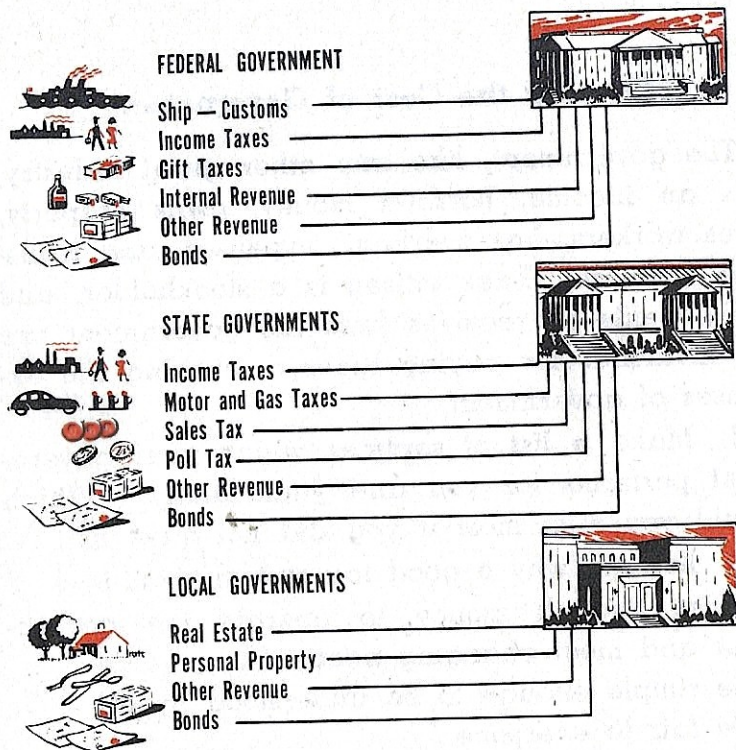
Be fair to everyone

Not be overlapping

3. Study the graph at the top of page 276. Can you think of any taxes that come under the headings **Other Revenue**?

Property Taxes. Property is listed as real and personal. **Real property** includes land and buildings. **Personal property** includes movable property such as furniture, automobiles, jewelry, stocks, bonds, or similar possessions.

Most local governments get the larger part of their taxes from the property tax. Officers called **assessors** place a value on property for tax purposes. This is called the **assessed value**.



Sources of Government Income

Each locality decided how much must be charged per \$100 worth of taxable property to raise sufficient taxes. The amount charged per \$100 is called the **tax rate**.

1. In the city of Redwood the tax rate is \$3 on each \$100 of assessed value. What is the tax on property assessed at \$4000?

Think: $\$4000 \div \$100 = 40$
 $\$3 \times 40 = \120 tax

2. Find the taxes on Redwood property assessed at: \$2,000 \$3,500 \$10,000 \$75,000 \$4,500 \$125.

Other Ways of Expressing Tax Rates

1. At \$2.80 per \$100, what is the tax on \$2,500 \$5,000 \$10,000 \$50 \$250 \$800

Sometimes the tax rate is expressed as a fractional per cent: \$2.80 per \$100 is the same as 2.8% (.028) of \$100.

2. Clay City has a tax rate of 2.8%. Find the tax on property assessed at—

\$10,000 \$3,000 \$25 \$150 \$9,000 \$750

Sometimes the tax rate is given in the number of mills paid on a dollar, as 3¢ on \$1 is read 30 mills on \$1.

3. How many mills are there in 1¢? In \$1?
4. Write as decimals, as 1 mill = \$.001:
 13 mills 6 mills 49 mills 2 mills
5. What tax will be paid on \$800 if the rate is 45 mills on \$1?
6. What tax does the Atlas Electric Store pay on a building assessed at \$54,000 and a stock of goods assessed at \$30,000 if the rate is $22\frac{1}{2}$ mills on \$1?
7. Mr. Sanders owns two houses. One is assessed at \$6,000 and the other at \$4,500. What is the tax on these houses if the rate is 35 mills on \$1?
8. Mr. Sanders listed his personal property as an automobile at \$600, furniture at \$800, and jewelry at \$225. What will his personal tax be at 35 mills on \$1?

Sales Tax

Many states and some cities raise money by charging a sales tax on goods sold to retail customers. If you have a sales tax in your state or town, compare your plan with any given here.

1. If a state or town has a 2% sales tax, how many mills are paid on \$1? On 5¢? On 10¢? On 40¢? On 65¢?

2. A man bought an automobile tire for \$10 and paid a 2% sales tax. What was the tax? The total cost of the tire?

One state has tax tokens. A red token is worth 1 mill and a green one 5 mills. The tax rate is 2%. Mills are charged on purchases of 4¢ or more. Find the amount of money and tokens a customer will receive as change from:

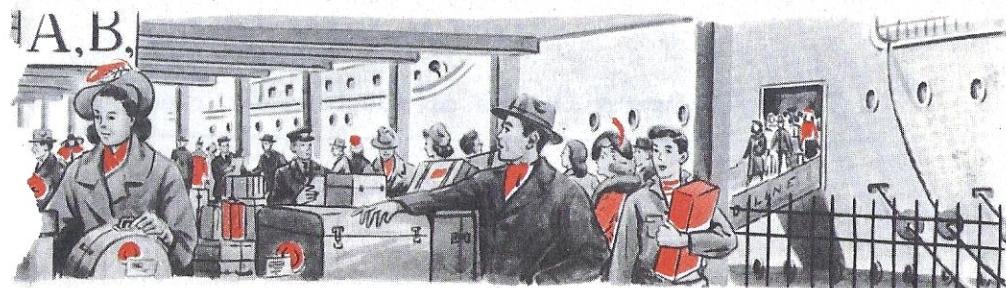
	Amount of Purchase	Money Handed to Clerk
3.	\$.26	\$.30
4.	\$.47	\$.50
5.	\$2.30	\$2.35

A certain state with a 2% sales tax collects the tax according to this table:

From \$.00 to \$.13	No tax
From \$.13 to \$.62	\$.01 tax
From \$.63 to \$ 1.12	\$.02 tax
From \$ 1.12 to \$ 1.62	\$.03 tax

Use the schedule above and find sales tax on the following purchases:

6. 15¢ 60¢ 98¢ \$1.45 7¢ 12¢



Duties or Customs

Part of the income of the United States comes from taxes paid on goods imported into this country. These taxes are called **import duties, customs taxes, or tariff.**

Every port of entry into the United States has a Custom House where duties are collected. Bringing goods into the country without paying proper duty is called smuggling. However, a resident of the United States returning from a foreign country is allowed to bring in \$100 worth of goods for his own use without paying duty.

Duties, or customs taxes, may be a certain per cent of the cost of the goods or a set amount may be charged by the pound, yard, or other unit of measure.

Duty charged according to value is called **ad valorem duty**. Duty charged on a fixed or specific amount is called **specific duty**.

1. Mr. Main imported 150 yards of velvet costing \$5.75 a yard. He paid 70% ad valorem duty. Find the cost of the velvet. Find the duty.

2. What would be the specific duty on a dozen pairs of gloves at \$2 duty per pair?

Table Showing Duties on Certain Articles

Article Imported	Specific Duty	Ad Valorem Duty
Automobiles	none	25%
Butter	8¢ a lb.	none
China	10¢ a doz. and	70%
Linens	none	45%
Rugs	none	55%
Safety Razors	10¢ each and	50%
Toys	none	70%

Use rates above and find the duty on:

1. A shipment of toys valued at \$3000.
2. Two thousand safety razors valued at 30¢ each.
3. Three thousand pieces of china valued at \$1500.
4. A shipment of 5 rugs valued at \$150 each.
5. A Persian rug valued at \$1500.
6. Mrs. West brought home \$60 worth of linens from Ireland and paid no duty. Why?

Highway taxes are collected from vehicle licenses and sales of gasoline. The gasoline tax varies from state to state, but the average tax is about 4.6¢ a gallon. The money collected from highway taxes is used to build and repair roads.

7. In one year \$700,000,000 was collected in the United States from gasoline taxes and \$60,000,000 from vehicle licenses. The gasoline tax is how many times as much as the licenses?

8. A truck driver bought $16\frac{1}{2}$ gal. of gasoline, tax $4\frac{1}{2}$ ¢ a gallon. What was the tax?

Income Taxes

The Federal Government and about three-fourths of the separate states raise a large part of their revenue from a tax on income. **Income taxes** are collected from individuals, both citizens and aliens, and from corporations.

Each year corporations and persons coming under the income tax law are required to file an income tax return with the government. This return must give a full account of the **total** or **gross income** received during the year, and state any **exemptions** or **deductions** allowed by law. When the **exemptions** and **deductions** are taken from the gross earnings, the remainder is the **net taxable income**.

From year to year laws governing income taxes are likely to change. Get a federal income tax blank for the present year and find the answers to the following.

1. What personal exemption or allowance is a single person permitted to take from his gross earnings?
2. What is the personal exemption of the head of a family?
3. What exemption is allowed for each dependent child?
4. Can a person deduct or take from the gross income money given to charity? To a church? Losses by fire not covered by insurance?

Problems About Income Tax

1. Mr. Barr earned \$4,500 last year and received \$35 a month rent for a house that he owns. What is his gross income?

2. Miss Horner earns \$1,500 a year. She has no dependents, but is allowed \$500 personal exemption. What is her net taxable income if she has no other deductions?

3. Miss East earned \$1,250 in salary last year and \$750 in commissions. What is her net taxable income if the government allows her \$1000 personal exemption and \$175 deductions for charity and a theft not covered by insurance?

4. Mr. Hart has a wife and three children. What is his net taxable income if his gross income is \$3,500 and he is allowed \$1,200 as head of the family and \$300 for each child?

5. What tax will Mr. Hill pay on a net taxable income of \$780 if the tax rate on amounts under \$4,000 is 2% of the net income?

6. Mr. Hays has a wife and two dependent children. He earned \$2,500 last year and had no other income. What is his tax if he is allowed \$1,200 as head of the family, \$250 for each child, \$125 deductions, and then pays 2% on his net taxable income?

Find out what is meant by a **surtax** on net incomes. Is the system of charging surtaxes fair? Explain your answer.

The Tip-Off

At \$2.85 per \$100 assessed valuation, find the tax on property assessed at:

1. \$7,000 \$3,200 \$15,000 \$25 \$450

2. Find the taxes on the property assessments given above when the rate of taxation is 3.1% of the assessed valuation.

3. Mr. Ames uses an average of 900 gallons of gasoline in a year. There is a tax of 5.1¢ collected on each gallon of gasoline sold in his state. How much tax does the state collect from the gasoline he uses in a year?

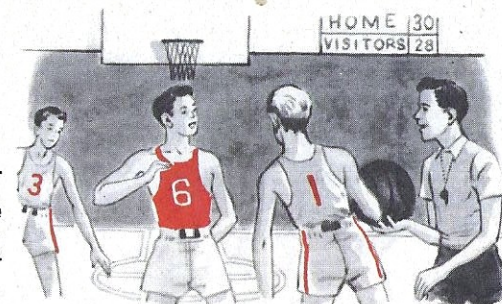
4. What is the duty on \$2,500 worth of toys if the importer pays 70% ad valorem duty?

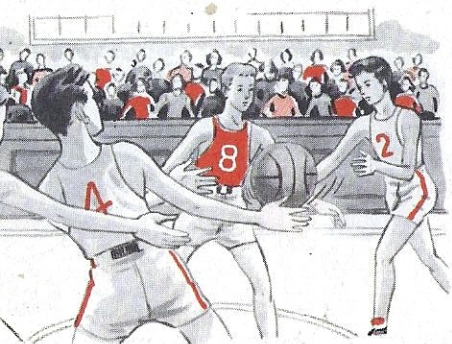
5. Miss Ware took out a 20-payment life insurance policy at an annual premium of \$26.58 per \$1000. If the face of the policy was \$5,000, what premium did she pay each year? What did she pay in 20 payments?

A certain state has a 2% sales tax and uses tokens to collect the tax to the nearest mill. What will be the tax on the following purchases?

6. 42¢ 36¢ \$1.06 \$3.24 \$56 \$800

If you had less than 18 correct answers on this page, work page 284. Other pupils will train as quiz masters on page 285.





Recovering the Ball

Work with a partner. When either of you needs assistance, look for help on the page given in parenthesis after the problem.

Find the taxes on property assessed at \$6,500 when the rate of property tax is:

1. \$3.50 per \$100 assessed valuation. (276)
2. 2.8% of the assessed valuation. (277)
3. 37 mills on \$1 valuation. (277)
4. \$20 per \$1000 assessed valuation. (277)

Write as decimals: (277)

5. 3 mills 15 mills 49 mills 2 mills

Find the yearly premium on the following policies using the table of rates given in this unit. (266)

6. Ordinary life for \$1,000, age 22.
7. Ordinary life for \$5,000, age 27.
8. 20-payment life for \$2,000, age 30.
9. 20-yr. endowment for \$3,000, age 24.
10. 10-yr. term for \$10,000, age 29.

Use the table of duties given in this unit and find the tariff on: (280)

11. \$500 worth of linen towels.
12. 8,000 lb. of butter.
13. 1,200 pieces of china worth \$1,800.
14. 600 safety razors at \$.35 each.

Passing the Ball

Train to be a quiz master! Study until you know the meaning of each of the terms listed below. Study the page given in parenthesis when you need help.

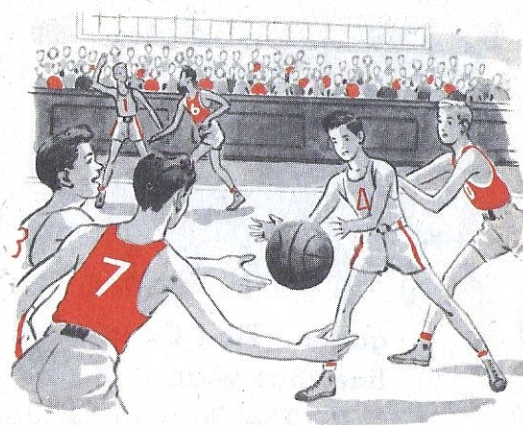
Then form a group of five pupils from those who are now working page 284. Quiz this group as, "What is meant by a 20-year endowment policy?" If a term is not known to any of the group, explain it yourself.

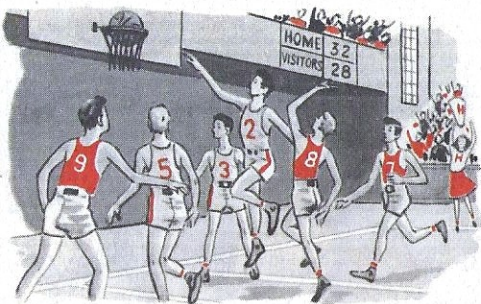
Terms used in insurance:

20-year endowment (265)	policy (262)
term insurance (264)	dividends (268)
ordinary life (265)	beneficiary (262)
cash surrender value (269)	loan value (270)
extended insurance (269)	premium (262)
paid-up insurance (269)	insured (262)

Terms used in taxes:

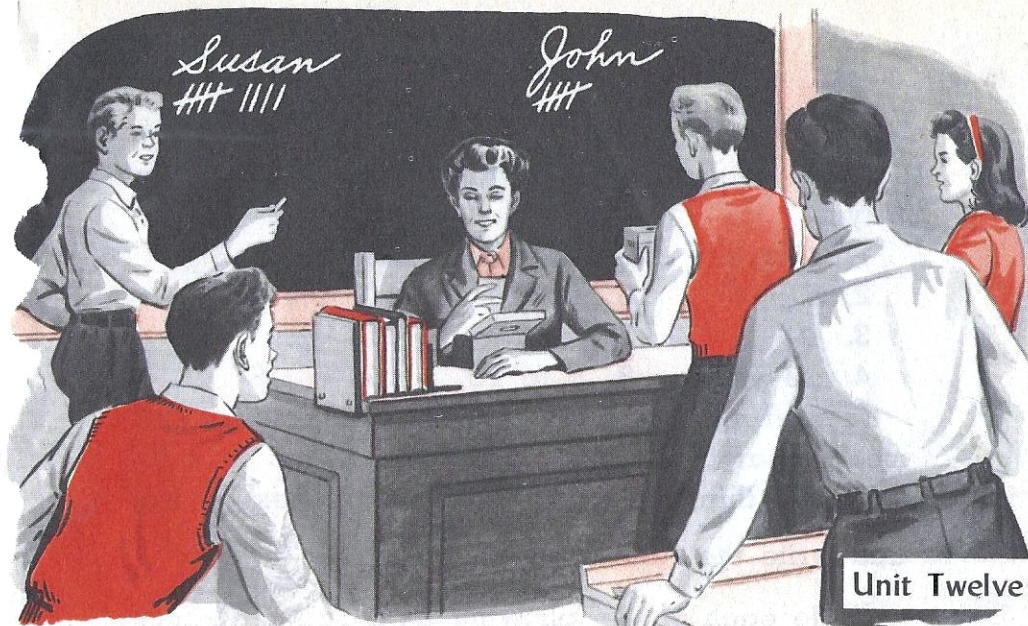
assessor (275)	property taxes (275)
sales tax (278)	custom taxes (279)
income tax (281)	assessed value (275)
duty (279)	specific duty (279)
taxes (275)	personal exemption (281)
gross income (281)	smuggling (279)
tariff (279)	custom house (279)





Goal for Unit Eleven

1. The State of Arizona sold 8,225,000 gallons of gasoline in a recent year. The state tax is 5¢ per gallon. Find the amount of tax collected on gasoline that year.
2. The State of Oklahoma collected \$1,183,280 in gasoline taxes in a single year. If the tax rate is 4¢ on a gallon, how many gallons were sold?
3. Find the tax on property assessed at \$4,000 when the rate is 32 mills on a dollar.
4. At the rate of \$2.90 on each \$100, what is the tax on an old car assessed at \$150?
5. Mr. Wills took out a \$5,000 20-payment life insurance policy paying \$27.71 annual premium per \$1,000. What had he paid on the policy after the last premium was paid? What sum of money would his beneficiary receive if he died after 5 payments were made? After 20 payments were made?
6. Miss Smith carries a \$1,000 ordinary life policy on which she pays an annual premium of \$22.94. Her company pays her an average of \$3.88 a year in dividends. What is the net yearly cost for this protection?
7. Find the duty on 150 cameras valued at \$75 each when the duty charged is 45% ad valorem.



Comparing Numbers by Division

In a class of 24 pupils, Susan and John had been candidates for president. Susan received 16 votes and John received 8 votes. The ratio of Susan's votes to John's was 16 to 8, or 2 to 1.

The relationship of two like numbers expressed as a fraction or a quotient is called a ratio.

The ratio 16 to 8 may be written as $\frac{16}{8}$, $\frac{2}{1}$, or 2. To find the ratio of two like numbers, write them as a fraction. Ratios expressed as fractions are usually reduced to lowest terms.

The ratio of 6 to 8 may be written $\frac{6}{8}$ or $\frac{3}{4}$.

The colon may be used as the sign of ratio. With the colon, the ratio 6 to 8 is written 6 : 8, and is read "6 to 8."

Using Ratios

Write each of the following ratios in two other ways.

- | | | |
|------------------|------------------|--------------------|
| 1. 4 to 6 | 5. 2 to 8 | 9. 12 to 8 |
| 2. 5 : 8 | 6. 10 : 4 | 10. 9 : 4 |
| 3. $\frac{4}{8}$ | 7. $\frac{6}{4}$ | 11. $\frac{2}{10}$ |
| 4. 12 : 16 | 8. 5 to 10 | 12. 3 : 9 |

In writing ratios, we use like numbers. This means that in writing the ratio of 24 in. to 3 ft., we must think of each as either inches or feet and write the ratio as $\frac{2 \text{ ft.}}{3 \text{ ft.}}$ or as $\frac{24 \text{ in.}}{36 \text{ in.}}$

Write each of the following as a ratio. Reduce to lowest terms.

- | | |
|--------------------|---------------------|
| 13. 6 ft. to 3 yd. | 16. 8 in. to 4 ft. |
| 14. 6 pt. to 4 qt. | 17. 4 da. to 2 wk. |
| 15. 4 oz. to 3 lb. | 18. 18¢ to 5 dimes. |

Draw a line 2 in. long and then draw lines having the following ratios to it.

- | | | | |
|-------------------|-------------------|-------------------|-------------------|
| 19. $\frac{1}{2}$ | 21. $\frac{3}{2}$ | 23. $\frac{1}{4}$ | 25. $\frac{3}{4}$ |
| 20. $\frac{2}{1}$ | 22. $\frac{3}{1}$ | 24. $\frac{1}{8}$ | 26. $\frac{3}{8}$ |

27. A hall rug is 48 in. wide and 12 ft. long. What is the ratio of its width to its length?

28. Rose weighs 72 lb. and Dick weighs 120 lb. What is the ratio of Rose's weight to Dick's weight?

29. The school team won 12 games and lost 3. What is the ratio of games won to games lost? Find the ratio of the games won to the total number of games.



Ratio in the Kitchen

Betty and Billy are preparing for a barbecue at which they plan to have a tossed salad and hamburgers. The recipes they have do not provide the quantities they need. They must double the recipe for barbecue sauce and halve the recipe for salad dressing.

- Write this recipe in the ratio of 2 : 1.

Barbecue Sauce

- | | |
|--------------------------------|-------------------------------|
| 1 8-oz. can tomato sauce | $\frac{1}{2}$ pt. water |
| $\frac{1}{4}$ c. chopped onion | $\frac{1}{2}$ pt. vinegar |
| 1 tbsp. brown sugar | $2\frac{1}{2}$ tbsp. salt |
| 1 tbsp. Worcester sauce | $\frac{1}{2}$ tsp. bl. pepper |

- Write this recipe in the ratio of 1 : 2.

Salad Dressing

- | | |
|----------------------------|------------------------|
| 1 10-oz. can tomato soup | 1 c. vinegar |
| $\frac{1}{2}$ tsp. paprika | 2 tsp. salt |
| $\frac{1}{2}$ tsp. pepper | $\frac{1}{3}$ c. sugar |
| 1 tsp. dry mustard | 1 tsp. onion juice |
| 1 clove of garlic | |

Pairs of Ratios

Two ratios are equal if they can be reduced to the same fraction.

Which of these pairs of ratios are equal?

- | | | |
|--------------------------------|--------------------------------|---------------------------------|
| 1. $\frac{2}{3}, \frac{4}{6}$ | 4. $\frac{8}{4}, \frac{2}{1}$ | 7. $\frac{3}{6}, \frac{9}{12}$ |
| 2. $\frac{1}{2}, \frac{8}{16}$ | 5. $\frac{5}{10}, \frac{4}{8}$ | 8. $\frac{4}{5}, \frac{12}{15}$ |
| 3. $\frac{3}{8}, \frac{6}{16}$ | 6. $\frac{3}{9}, \frac{9}{18}$ | 9. $\frac{6}{9}, \frac{4}{6}$ |

When two equal ratios are set equal to each other, they form a **proportion**.

$\frac{2}{3} = \frac{4}{6}$ is a proportion. You read this proportion as "2 is to 3 as 4 is to 6."

10. Write in fraction form as the equality of two ratios: 4 is to 16 as 8 is to 32.

Copy. Supply the missing numbers.

- | | | |
|----------------------------------|----------------------------------|----------------------------------|
| 11. $\frac{?}{6} = \frac{2}{3}$ | 13. $\frac{?}{6} = \frac{8}{12}$ | 15. $\frac{?}{5} = \frac{8}{2}$ |
| 12. $\frac{?}{12} = \frac{4}{8}$ | 14. $\frac{?}{9} = \frac{2}{6}$ | 16. $\frac{?}{12} = \frac{6}{3}$ |

A *proportion is an equation*. You solve for an unknown letter in a proportion as you solve for the value of a letter in any equation.

Solve the following equations for x .

- | | | |
|------------------------|-----------------------|-----------------------|
| 17. $\frac{1}{2}x = 4$ | 20. $\frac{x}{2} = 4$ | 23. $\frac{x}{3} = 6$ |
| 18. $\frac{1}{4}x = 2$ | 21. $\frac{x}{4} = 2$ | 24. $\frac{x}{4} = 4$ |
| 19. $\frac{1}{6}x = 3$ | 22. $\frac{x}{6} = 3$ | 25. $\frac{x}{8} = 5$ |

Using Proportions

1. How many sixths are equal to two thirds?

Let x stand for the required number of sixths, and write the proportion:

$$\frac{x}{6} = \frac{2}{3}$$

Think of this proportion as an equation. What must you multiply the left side of the equation by to have x alone? You must multiply the right side by this same number.

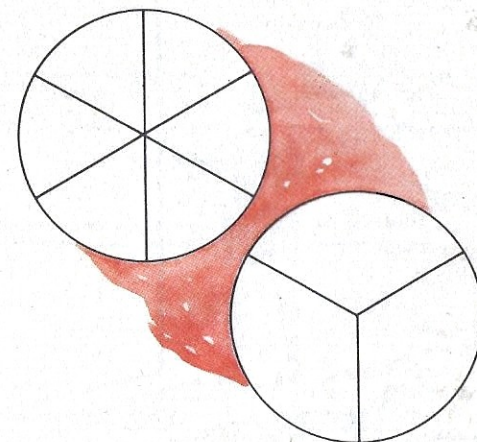
$$6 \times \frac{x}{6} = 6 \times \frac{2}{3}, \quad \frac{6x}{6} = \frac{12}{3}, \quad x = 4$$

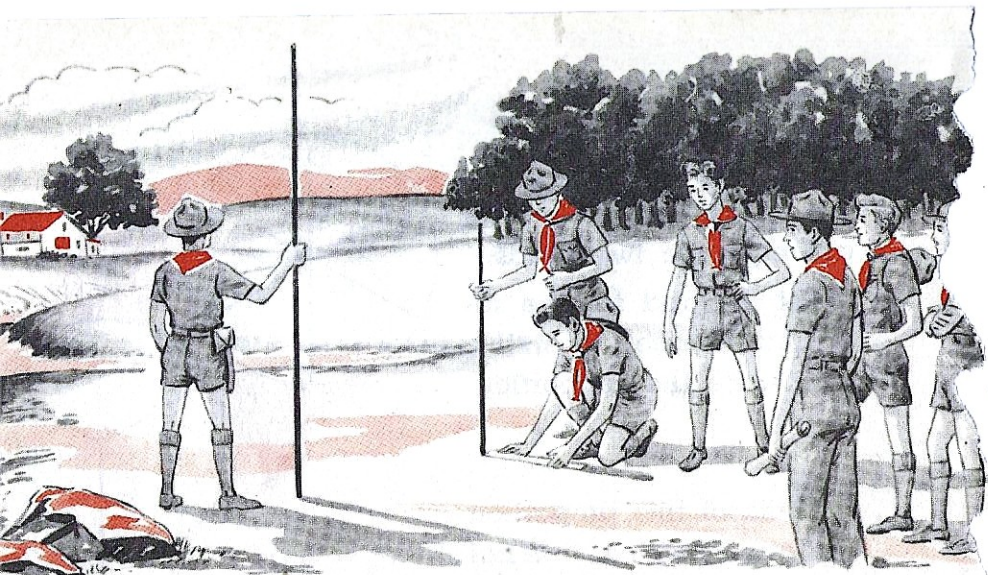
It takes 4 sixths to equal 2 thirds.

Solve the following.

- | | | |
|---------------------------------|------------------------------|------------------------------|
| 2. $\frac{x}{12} = \frac{3}{4}$ | $\frac{x}{12} = \frac{1}{3}$ | $\frac{x}{12} = \frac{2}{3}$ |
| 3. $\frac{x}{6} = \frac{1}{3}$ | $\frac{x}{8} = \frac{1}{4}$ | $\frac{x}{6} = \frac{1}{2}$ |
| 4. $\frac{x}{15} = \frac{2}{5}$ | $\frac{x}{15} = \frac{2}{3}$ | $\frac{x}{15} = \frac{4}{5}$ |
| 5. $\frac{x}{6} = \frac{1}{4}$ | $\frac{x}{2} = \frac{3}{4}$ | $\frac{x}{5} = \frac{3}{10}$ |

6. Let x stand for Elsie's earnings. Her father earns \$180 a month. Set up and solve the proportion: Elsie's earnings are to father's earnings as 7 is to 12.





Finding Heights with Shadows

The Scoutmaster told his troop that it was possible to find the height of an upright object by measuring its shadow. He asked Bill and John to cut two poles, one 8 ft. long and the other 4 ft. long. He had the boys hold the poles upright on a level spot in the sunshine. Andrew measured the lengths of the shadows. The shadow of the 8-foot pole was 6 ft. long, and the shadow of the 4-foot pole was 3 ft. long. The Scoutmaster explained that the heights of upright objects are in the same ratio as their shadows.

$$\frac{8}{4} = \frac{6}{3}$$

The Scoutmaster then pretended he did not know the height of the taller pole and showed the boys how they could find it. He let x stand for its height and wrote the proportion, $\frac{x}{4} = \frac{6}{3}$ and solved it for x .

The boys had fun solving these problems. Try them.

1. A pole 10 ft. tall casts a shadow 6 ft. long at the same time a tree casts a shadow 21 ft. long. How tall is the tree?

2. A factory smokestack casts a shadow 120 ft. long at the same time a 10-foot pole casts a shadow 8 ft. long. How tall is the smokestack?

3. John's father is 6 ft. tall. At a certain time of day his shadow is 4 ft. long. At the same time, the shadow of John's house is 18 ft. long. How high is the house?

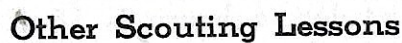
4. A telephone pole casts a shadow 12 ft. long at the same time an 8-foot pole casts a shadow 3 ft. long. How tall is the telephone pole?

5. Bill and John wondered how tall the camp flagpole was. They found that it cast a shadow 15 ft. long at the same time an 8-foot pole cast a shadow of 2 ft. How tall was the flagpole?

6. John is 5 ft. tall. At a certain time of day his shadow was 2 ft. long and the shadow of a tree was 24 ft. long. How tall is the tree?

7. The shadow of a garage was 7 ft. long at the same time the shadow of the house was 21 ft. The garage was 10 ft. high. How high was the house?

8. The Scoutmaster suggested they find Andrew's height by comparing his shadow with the shadow of a yardstick. Andrew's shadow was 30 in. when the shadow of the yardstick was 18 in.



- Sight from point A on the river bank to a point B directly across on the opposite bank.
- Lay off AC to make a right angle with AB .
- Lay off CD to make a right angle with AC .
- Sight from D to B , and drive a stake at E where this line of sight crossed AC .

- e. Measure distances AE , EC , and CD .

The Scoutmaster told the boys to think of AE and EC as like shadows, and AB and CD as like heights. He then wrote the proportion, $\frac{x}{CD} = \frac{AE}{EC}$

2. Use this method to find the distance between two trees on opposite sides of a street.

Distances on scale drawings and maps are in proportion to the real distances between points. The scale of the drawing or map gives the ratio. The scale of the map at the right is $1'' = 20 \text{ mi.}$

Let x equal the real distance in miles. Then x is to the map distance as 20 is to 1.

Find the distance from
Camp Crossbones to

3. Roaring River
4. Echo Ridge
5. Indian Lake
6. Arrowhead Hill
7. Blind Fish Caverns

8. A baseball diamond is 90 ft. square. The pitcher's box is 60 ft. from home plate. Make a scale drawing of the diamond, using the scale, $1'' = 15'$.



Ratios in Problems

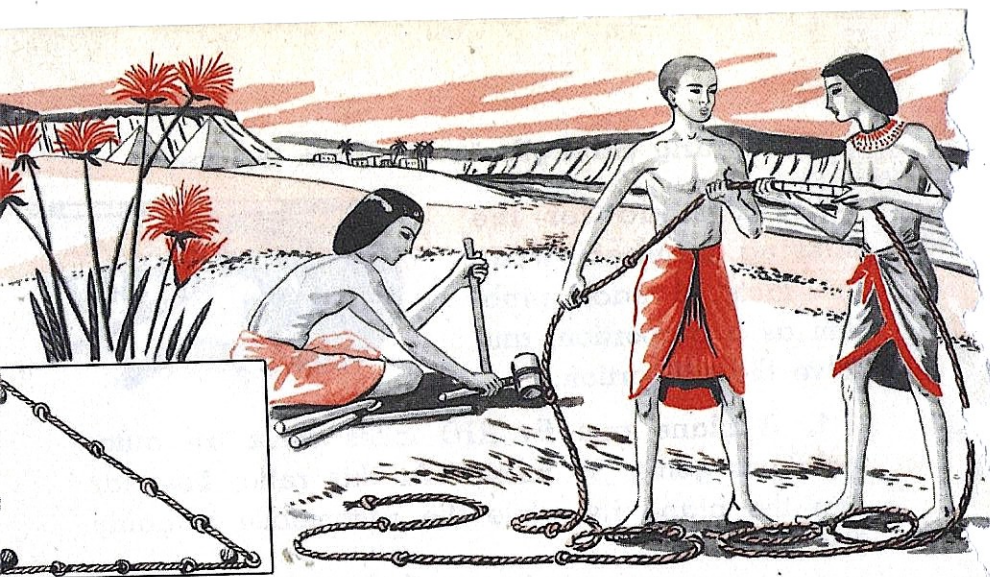
1. Jack earns \$48 a week and spends \$40. What is the ratio of his spendings to his earnings? What is the ratio of his savings to his spendings?
2. Jim's bicycle cost \$32 and Sam's cost \$28. What is the ratio of the cost of Jim's bicycle to Sam's? What is the ratio of the cost of Sam's bicycle to Jim's?
3. Mrs. Jones bought a 75-dollar chair for \$60. What part of the regular price did she pay?
4. A farmer mixed 30 lb. of corn, 20 lb. of oats, and 50 lb. of bran for feed. What part of the mixture was corn? What part was oats? What part was bran?
5. In Problem 4, find the ratio of corn to oats; of oats to bran; of bran to corn.
6. It takes 108 gal. of maple sap to make 24 lb. of maple sugar. How many gallons of sap are needed to make 132 lb. of sugar?
7. In making a disinfectant, a farmer uses one part of stock dip and 48 parts of water. How much stock dip will he mix with 24 gal. of water?
8. In Problem 7, how many gallons of water would the farmer use with 1 pt. of stock dip?
9. Mrs. Smith gets 40 eggs per day from 60 hens. At the same rate, how many eggs should Mrs. Brown get per day from the 84 hens she has?

Gaining Strength in Problem Solving

Let x stand for the missing number, write the facts of each problem as a proportion, and solve the proportion.



1. A plane can fly 210 miles while an automobile is going 35 miles. At this ratio, how far can the plane fly while the automobile is going 105 miles?
2. A ball team won 5 games out of 8. At this rate, how many games will they win out of 64 they plan to play?
3. Ice cream for 8 cost 60 cents. At this rate, what is the cost of ice cream for 14?
4. Mr. Allen worked 13 days last month and earned \$91. He worked 23 days this month. What should his earnings be?
5. How many teachers should be employed in a school having 1,250 pupils if a school having 1,000 pupils employs 36 teachers?
6. Jackson School serves lunches to 150 pupils at a cost of \$24 a day. What should be the cost per day of serving lunch to 225 pupils?
7. How many copies of a newspaper can be printed in $2\frac{1}{2}$ hours if 60,000 copies can be printed in $1\frac{1}{2}$ hours?



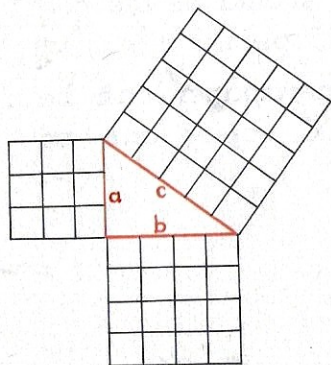
The Pythagorean Rule

Surveyors of ancient Egypt were called rope stretchers. They were able to make a right angle by use of the triangle of knotted rope as shown above. Notice that the altitude is 3 units long, the base is 4 units long, and the **hypotenuse** (side opposite the right angle) is 5 units long.

Pythagoras, a Greek mathematician, knew of the triangle of knotted rope the Egyptians used. He proved this rule true for all right triangles: *The square on the altitude plus the square on the base is equal to the square on the hypotenuse.*

To understand this rule, study the figure at the left. This rule can be written as a formula,

$$a^2 + b^2 = c^2$$



Using the Rule of Pythagoras

You need squares and square roots of numbers in using the Rule of Pythagoras. The ones you will need on this page are given in the table at the right. The square of each number in Column A is across from it in Column B. The square root of each number in Column B is across from it in Column A.

The **square** of any number is found by multiplying the number by itself. The square of 4 is 16.

The **square root** of a given number is a number which when multiplied by itself will have the given number as a product. The square root of 16 is 4.

- Find the squares of 17, 26, 13, 9.
- Find the square roots of 225, 625, 49, 169.

Use $c^2 = a^2 + b^2$ and find c when —

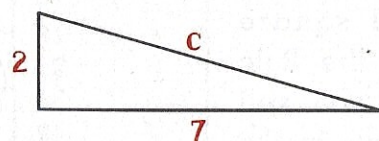
- $a=3, b=4$
- $a=7, b=24$
- $a=5, b=12$
- $a=8, b=6$
- $a=8, b=15$
- $a=24, b=10$

Use $a^2 = c^2 - b^2$ and find a when —

- $c=13, b=12$
- $c=15, b=12$
- $c=26, b=24$
- $c=25, b=24$
- $c=25, b=15$
- $c=17, b=8$

A	B
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
12	144
13	169
15	225
17	289
20	400
24	576
25	625
26	676

Squares and Square Roots



The altitude of a right triangle is 2, the base is 7. Then $c^2 = 4 + 49 = 53$, and c is the square root of 53.

You can find the approximate square root of 53 in the table on page 301. Look in the column marked "Number" until you find 53. Look to the right in the column marked "Square Root." There you find 7.280. This is the approximate square root of 53. Then $c = 7.280$.

1. Use the table on page 301 to find the square roots of 27, 46, 65, 76, 94, 103, 119.

The table on page 301 will give you the squares as well as the square roots of numbers.

2. Use the table on page 301 to find the squares of 64, 37, 45, 92, 17, 112, 86.

Find c in each of these right triangles.

- | | |
|-------------------|--------------------|
| 3. $a = 1, b = 5$ | 9. $a = 3, b = 5$ |
| 4. $a = 3, b = 2$ | 10. $a = 8, b = 1$ |
| 5. $a = 2, b = 6$ | 11. $a = 6, b = 7$ |
| 6. $a = 4, b = 5$ | 12. $a = 9, b = 3$ |
| 7. $a = 5, b = 6$ | 13. $a = 7, b = 7$ |
| 8. $a = 7, b = 3$ | 14. $a = 5, b = 9$ |

TABLE OF SQUARES AND SQUARE ROOTS

Number	Square	Square Root	Number	Square	Square Root	Number	Square	Square Root
1	1	1.000	41	1681	6.403	81	6561	9.000
2	4	1.414	42	1764	6.481	82	6724	9.055
3	9	1.732	43	1849	6.557	83	6889	9.110
4	16	2.000	44	1936	6.633	84	7056	9.165
5	25	2.236	45	2025	6.708	85	7225	9.220
6	36	2.449	46	2116	6.782	86	7396	9.274
7	49	2.646	47	2209	6.856	87	7569	9.327
8	64	2.828	48	2304	6.928	88	7744	9.381
9	81	3.000	49	2401	7.000	89	7921	9.434
10	100	3.162	50	2500	7.071	90	8100	9.487
11	121	3.317	51	2601	7.141	91	8281	9.539
12	144	3.464	52	2704	7.211	92	8464	9.592
13	169	3.606	53	2809	7.280	93	8649	9.644
14	196	3.742	54	2916	7.348	94	8836	9.695
15	225	3.873	55	3025	7.416	95	9025	9.747
16	256	4.000	56	3136	7.483	96	9216	9.798
17	289	4.123	57	3249	7.550	97	9409	9.849
18	324	4.243	58	3364	7.616	98	9604	9.899
19	361	4.359	59	3481	7.681	99	9801	9.950
20	400	4.472	60	3600	7.746	100	10,000	10.000
21	441	4.583	61	3721	7.810	101	10,201	10.050
22	484	4.690	62	3844	7.874	102	10,404	10.100
23	529	4.796	63	3969	7.937	103	10,609	10.149
24	576	4.899	64	4096	8.000	104	10,816	10.198
25	625	5.000	65	4225	8.062	105	11,025	10.247
26	676	5.099	66	4356	8.124	106	11,236	10.296
27	729	5.196	67	4489	8.185	107	11,449	10.344
28	784	5.292	68	4624	8.246	108	11,664	10.392
29	841	5.385	69	4761	8.307	109	11,881	10.440
30	900	5.477	70	4900	8.367	110	12,100	10.488
31	961	5.568	71	5041	8.426	111	12,321	10.536
32	1024	5.657	72	5184	8.485	112	12,544	10.583
33	1089	5.745	73	5329	8.544	113	12,769	10.630
34	1156	5.831	74	5476	8.602	114	12,996	10.677
35	1225	5.916	75	5625	8.660	115	13,225	10.724
36	1296	6.000	76	5776	8.718	116	13,456	10.770
37	1369	6.083	77	5929	8.775	117	13,689	10.817
38	1444	6.164	78	6084	8.832	118	13,924	10.863
39	1521	6.245	79	6241	8.888	119	14,161	10.909
40	1600	6.325	80	6400	8.944	120	14,400	10.954

Using the Pythagorean Rule in Problems

1. A 12-foot ladder leans against a wall so that the foot of the ladder is 5 feet from the wall. How high up the wall does the ladder reach?
2. A farm gate is 8 ft. long and 4 ft. high. How long is the brace that reaches from a lower corner to the opposite upper corner?
3. A suitcase is 28 in. long and 15 in. high. Will an umbrella 36 in. long fit in the suitcase? Prove your answer.
4. A tent pole is 8 ft. tall. How long should a tent rope be to reach from the top of the pole to a point 7 ft. from the base of the pole?
5. A screen door is 7 ft. by 4 ft. How long a brace is needed to reach from a lower corner to the opposite upper corner?
6. A card table is 3 ft. square. What is the length of a diagonal?
7. A gable roof has a rise of 5 ft. in a span of 12 ft. What is the length of the rafter?
8. A rectangular flower garden is 8 ft. by 15 ft. What is the length of a diagonal?
9. A state park is a rectangle 3 mi. long and 2 mi. wide. In going from a corner to the diagonally opposite corner, how much nearer is it to go along the diagonal than along two sides?



A Lesson in Government and Arithmetic

Thirty pupils studying government in the Midway School decided to take a trip to the state capital. Here are some of the problems they had to solve in planning the trip.

1. The fare one way was 75¢. At this rate what would 30 round trips cost?
2. A Grandview bus could be chartered for \$36 for the entire trip. What would each pupil save on a round trip at this rate?
3. The bus left Midway at 8:55 A.M. and reached the capital at 12:03 P.M. At this rate how long did a round trip take?
4. The chairman of the tour committee wrote his representative in the capital. The representative arranged a $1\frac{1}{2}$ -hour visit to a session of the state legislature; a 4-hour visit to state buildings; a 90-minute visit to a government class in one of the schools; and a tea from 4:30 to 6:00 P.M. to meet the governor and his wife. How long would the entire tour take? How long would the entire trip take, including time on the bus?
5. The class felt the trip could not be made in a day, and so plans were made for a day and a half. The class voted to take two mothers, a

school board member, the school nurse, and the teacher as guests. How many persons would go on the trip, including all of the class?

6. What would be the ratio of grown-ups to pupils on the trip? Was this plan wise?

7. The committee on expenses found that a double room at the best hotel cost \$1.50 per person, breakfast was \$.35, dinner \$.60, and lunch in the cafeteria of the school visited would be \$.25. What would rooms and these meals cost for the whole party?

8. The thirty pupils voted to share equally the cost of rooms, meals, and the bus fare for themselves and guests. What will each pay?

The class agreed to earn the money for the trip by working out money-making projects approved by their parents and teacher.

9. So each pupil could have some class-earned capital to finance his project, half of the class gave "The Life of Columbus," a puppet show. Sixty children paying 10¢ each and forty adults paying 25¢ each attended the show. What were the profits if expenses were \$2.35?

10. The rest of the class conducted weekly scrap paper collections. They collected and sold 9000 lb. of newspaper at \$.40 per hundred pounds and 4200 lb. of magazines at \$.60 per hundred pounds. What were the earnings from the collections of scrap paper?

Making Dollars Grow

1. Each pupil was given \$2.25 from the profits of the puppet show and scrap paper collection to invest in a money-making project. The rest of the profits were put in a communications fund for postage, telephone calls, or telegrams. How much was put in this fund?

2. Jean used her \$2.25 to buy $5\frac{1}{4}$ yd. of 16-inch linen toweling at 40¢ a yard, a 5¢ spool of thread, and a 5¢ package of needles. She paid a 2% sales tax, using cents and mill tokens. What did she have left from her \$2.25?

3. Jean cut the linen into pieces $\frac{3}{4}$ yd. long, putting hand-hemstitched hems on the ends of each towel. Her aunt bought them at \$.79 each when they were finished. To what sum had Jean's \$2.25 grown?

4. The towels were to be 24 in. long when finished. How much did Jean use for each hem? Draw a pattern showing the dimensions of the towel and hems, using $\frac{1}{4}$ " to represent 3"

5. Her aunt wanted the towels packed individually in gift boxes. Jean folded the finished towels in thirds along the length and then in halves across the length. What is the smallest box she can use if the folded towel is $\frac{3}{4}$ in. thick?

6. How much did Jean put in the tour fund after she paid \$.35 for gift boxes, \$.05 for wrapping tissue, and 2¢ a yard for 14 yards of ribbon?

Building Profits

Don had an electric saw and tools. He and three other boys agreed to invest their \$2.25 in making hanging shelves for kitchens like the unit shown here.



1. The backing for the shelves is a piece of plywood $16'' \times 28\frac{2}{3}''$. How many backs can be cut from a sheet of plywood measuring $48'' \times 86''$?

2. The sheet of plywood costs \$4.95. What is the cost of material for one back?

3. The plywood backing extends $1\frac{2}{3}''$ above the top shelf. How long is the piece of lumber forming the right side of the unit?

4. How long are the shelves at the right if the lumber used is $\frac{1}{2}''$ thick?

5. The longest shelves measure $24''$. How many feet of lumber will it take to make the sides and the shelves?

6. What will the lumber cost at \$.06 a running foot?

7. It will cost \$.20 for wood trim and \$.05 to paint each unit. What will it cost to make the entire unit?

8. Don said their combined capital would buy material for six units if they bought a $48'' \times 60''$ sheet of plywood at \$3.30, instead of the \$4.95 piece. Was he right?

9. Would the $48'' \times 60''$ sheet make six backs? Draw a diagram of the cutting.

10. They finished and sold 12 units in all at \$2.50 each. What did they get for them? How much of this did they earn themselves? What were the average earnings per boy?

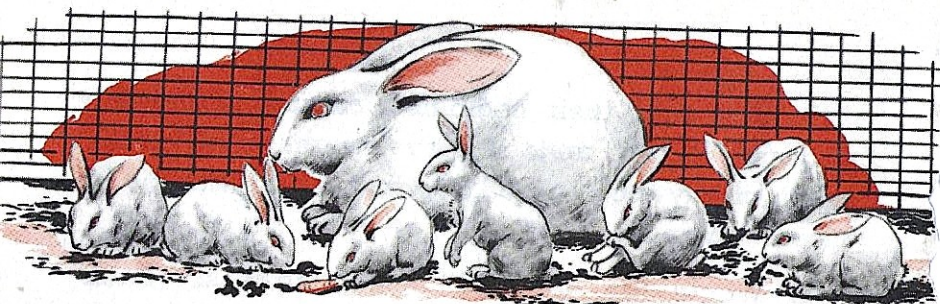
11. Will the average earnings plus the original capital allowed each boy be more or less than he needs for the trip?

Keeping a Record of Earnings

As pupils deposited earnings and capital with the tour treasurer, the treasurer and the teacher recorded the deposits in an account book and wrote them again on the blackboard for the class to check. Keep a record as they did, filling in deposits as you learn what they were from these problems.

Tour Fund Deposits

Names of Pupils	Projects	Capital	Earnings	Total
Jean	Towels	\$ 2.25	\$2.60	\$4.85
Don—others	Shelves	9.00		
Jim—others	Rabbits	15.75		



Dollars from Rabbits

Jim and six other boys of the class agreed to take care of 12 mother rabbits and their 94 little rabbits, and to use their capital to buy the grain and protein part of their food for eight weeks. In return they were to receive half the selling price of the 94 young rabbits.

The table gives the number of ounces of the grain-protein mixture each mother rabbit and each young rabbit will eat daily.

Week	Mother	Young
1 st	5.0	0
2nd	5.5	0
3rd	6.0	0
4th	6.0	.75
5th	6.0	1.50
6th	6.0	2.00
7th	6.0	2.50
8th	6.0	2.75

1. How many ounces of the mixture will a mother rabbit eat the first week? The second week? In the eight weeks?

2. How many pounds of the mixture will a mother rabbit eat in 8 weeks? How many pounds will the 12 mother rabbits eat in 8 weeks?

3. How many pounds of the mixture will the 94 young rabbits eat in 8 weeks?

4. How many pounds of the mixture will all of the rabbits eat in 8 weeks?

5. The grain-protein mixture in weight is to be about $\frac{1}{3}$ oats, $\frac{1}{3}$ wheat, and $\frac{1}{3}$ linseed meal. About how many pounds of each will the boys need to buy?

6. Jim figured that 7 bu. of oats (32 lb. per bushel), 4 bu. of wheat (60 lb. per bushel), and two 100-lb. bags of linseed meal would be enough. Did he figure correctly?

7. Find the total cost of the amounts listed in Problem 6 at these prices: wheat, 96¢ a bushel; oats, 45¢ a bushel; linseed meal, \$.04 a pound.

8. At the end of eight weeks the rabbits averaged $4\frac{1}{8}$ pounds each. The boys sold them at 32¢ a pound. How much did they get for them?

9. What was the boys' share of the money received for the young rabbits? What were the average earnings per boy after paying for the grain-protein mixture?

The boys learned these facts about rabbits:

10. The hutches should be $2\frac{1}{2}$ ft. wide and 2 ft. high. For small breed rabbits, the hutch should be 3 ft. long; for medium weight breeds, 4 ft. long; and for giant breeds, 6 ft. long. What amount of space (volume) is allowed for each size breed?

11. The dressed weight of a young rabbit is about 55% of its live weight. What is the dressed weight of a rabbit whose live weight is $4\frac{1}{8}$ pounds?

Other Projects

1. June worked in the school library from 3:30 to 4:45 each schoolday afternoon and from 9:00 to 11:45 on Saturday, earning 25¢ an hour. She pledged to give $\frac{1}{5}$ of her earnings for eight weeks. How much did she give? Was this enough to make her share, considering that she did not use her \$2.25 working capital?

Mary and Betty invested their capital in 12 qt. of strawberries, costing \$2.25. They used 9 cups of sugar to each 2 qt. of berries and made enough preserves to fill 22 pint jars.

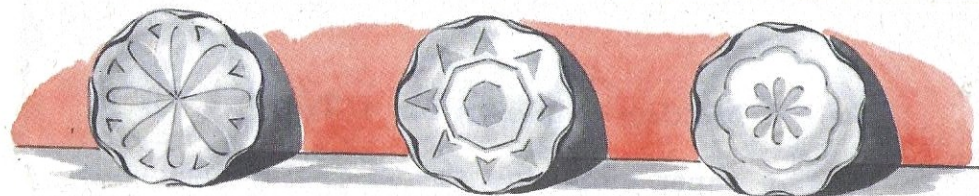
2. What did the sugar cost at 5¢ a pound if there are 2 cups to the pound?

3. What did 22 jars cost at \$.48 a dozen?

4. Did the two girls have enough capital together to buy the things needed for their project?

5. How much did the girls each earn if the preserves sold at \$.40 a pint?

6. Ted pledged $\frac{1}{5}$ of his earnings from his auto laundry for two weeks. How much did he give if he paid 98¢ for cleaner, and polished 2 cars at \$5 each and washed 6 at 60¢ each?



Making Aluminum Trays

1. The trays above were made from aluminum disks. Fourteen pupils agreed to make these trays for their money-making project. What was the combined capital allowed this group?

The bill for aluminum disks ordered by these pupils is shown below.

2. How many disks did they buy? How many disks were a foot or over in diameter?

3. Excluding the sales tax, find the cost of one 21-inch disk; of one 9-inch; 12-inch.

4. What was the rate of sales tax, counting the exact tax as \$.131?

METAL GOODS CORPORATION

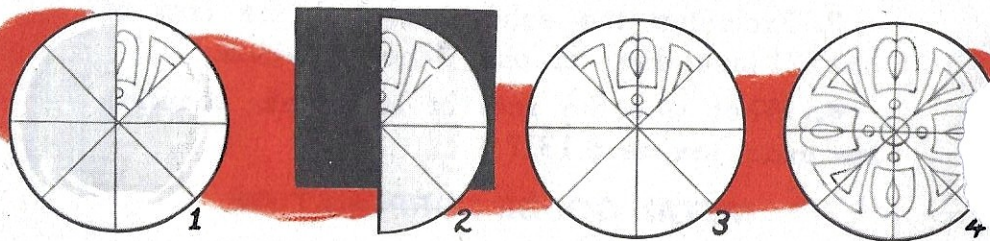
ALUMINUM ST. LOUIS, MO. STAINLESS STEEL
COPPER
BRASS CENTRAL 1280 ZINC ACCESSORIES
M Midway School 12/3/48
Address 3909 Pennsylvania

QUAN.	DESCRIPTION	PRICE	AMOUNT
6	21" Aluminum Disks 16 Gauge		\$ 3.54
6	14" " " 18 "		1.56
5	12" " " " "		.85
6	9" " " " "		.60
			6.55
	Sales Tax		.13
			\$ 6.68

Making a Design for a Tray

Cut a paper circle the size of your aluminum disk. Fold it into eight equal sectors. Sketch half of a unit of design along the radius of one sector (Figure 1). Complete the unit by retracing it after having placed a piece of carbon paper under the sector, carbon side up (Figure 2). Open your circle (Figure 3). If you like the unit you have completed, trace it in the other three quarters of the circle (Figure 4).

If you think your design is not good, begin again with a new paper circle.



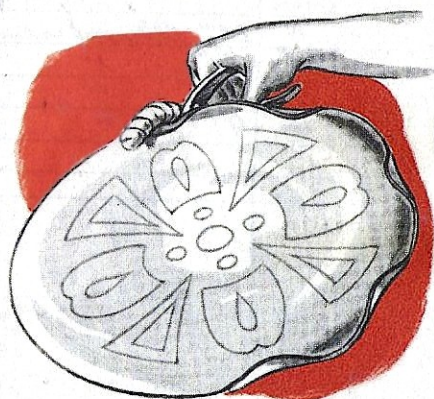
How to Make a Tray

Rub aluminum disk one-half hour in one direction with a circular motion, using No. 000 steel wool.

Trace design on disk, using carbon paper.

Turn up edges of disk $\frac{1}{2}$ " to $\frac{3}{4}$ " at eight points equally spaced around the circumference. Use pliers wrapped in cloth to prevent marring.

312



Paint with stove enamel everything that will be shiny when the tray is done. Dry over night.

Etch by pouring a solution of $\frac{1}{2}$ water and $\frac{1}{2}$ muriatic acid into tray.

When etched deep enough, pour off acid and wash tray in water.

Clean off enamel with turpentine. Then wash tray clean in water. Polish front and back of tray using steel wool as above.

1. Find the total cost of the following materials used in making the trays: \$6.68 for metal; 3 rolls of steel wool for each pupil at 2 rolls for \$.15; \$1.20 for muriatic acid; 10¢ each for 10 cans of stove enamel; \$.45 for carbon paper; \$.95 for turpentine; \$.65 for paint brushes.

2. The trays were sold at an average of \$3.25 each when finished. How much above expenses did the project make?

3. Balance the tour deposit record. How much above expenses for the trip is the total fund after all pupils have settled their accounts?

4. Which project produced the highest average earnings per pupil working on it? Which project produced the lowest average earnings per pupil?

313

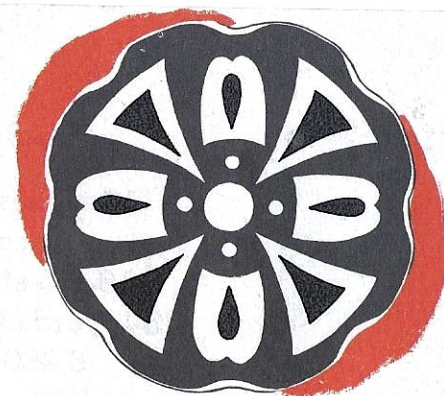


Table of Length

12 inches (in.) = 1 foot (ft.)
 3 feet (ft.) = 1 yard (yd.)
 $16\frac{1}{2}$ feet (ft.) } = 1 rod (rd.)
 $5\frac{1}{2}$ yards (yd.) }
 5,280 feet }
 1,760 yards } = 1 mile (mi.)
 320 rods (rd.) }

Table of Area or Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
 9 square feet (sq. ft.) = 1 square yard (sq. yd.)
 $30\frac{1}{4}$ square yards (sq. yd.) = 1 square rod (sq. rd.)
 43,560 square feet } = 1 acre (A.)
 160 square rods (sq. rd.) }
 640 acres (A.) = 1 square mile (sq. mi.)

Table of Volume or Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
 27 cubic feet (cu. ft.) = 1 cubic yard (cu. yd.)
 128 cubic feet (cu. ft.) = 1 cord (C.)

Table of Liquid Measure

2 cups = 1 pint (pt.)
 2 pints (pt.) = 1 quart (qt.)
 4 quarts (qt.) = 1 gallon (gal.)
 1 gallon (gal.) = 231 cubic inches (cu. in.)
 $7\frac{1}{2}$ gallons (gal.) = 1 cubic foot (cu. ft.)

Dry Measure

2 pt. = 1 qt.
 8 qt. = 1 pk.
 4 pk. = 1 bu.

Avoirdupois Weight

16 oz. = 1 lb.
 100 lb. = 1 cwt.
 2000 lb. = 1 T.

Measurements of Time

60 seconds (sec.) = 1 minute (min.)
 60 minutes (min.) = 1 hour (hr.)
 24 hours (hr.) = 1 day (da.)
 7 days (da.) = 1 week (wk.)
 30 days (da.) } = 1 business month
 4 weeks (wk.) }
 30 (29, 28, 31) days = 1 month (mo.)
 52 weeks (wk.) } = 1 year (yr.)
 12 months (mo.) }
 360 days (da.) = 1 business year
 365 days (da.) = 1 common year
 366 days (da.) = 1 leap year
 10 years (yr.) = 1 decade
 10 decades } = 1 century
 100 years }

Counting Things

12 things = 1 dozen (doz.)
 12 dozen (doz.) } = 1 gross (gro.)
 144 things }

Miscellaneous Measures

6 ft. = 1 fathom
 6080.20 ft. = 1 nautical mile
 3 nautical miles = 1 league
 2240 lb. = 1 long ton
 2150.42 cu. in. } = 1 bu.
 $1\frac{1}{4}$ cu. ft. }

The 100 Addition Facts

You know most of these facts now. Find those you miss, copy them, and practice until you are sure you know them.

1. $\begin{array}{r} 3 \\ 1 \end{array} \begin{array}{r} 3 \\ 3 \end{array} \begin{array}{r} 6 \\ 1 \end{array} \begin{array}{r} 0 \\ 7 \end{array} \begin{array}{r} 2 \\ 1 \end{array} \begin{array}{r} 1 \\ 1 \end{array} \begin{array}{r} 2 \\ 0 \end{array} \begin{array}{r} 0 \\ 0 \end{array} \begin{array}{r} 8 \\ 1 \end{array} \begin{array}{r} 2 \\ 2 \end{array}$
2. $\begin{array}{r} 0 \\ 3 \end{array} \begin{array}{r} 1 \\ 0 \end{array} \begin{array}{r} 0 \\ 9 \end{array} \begin{array}{r} 1 \\ 8 \end{array} \begin{array}{r} 6 \\ 0 \end{array} \begin{array}{r} 1 \\ 6 \end{array} \begin{array}{r} 0 \\ 4 \end{array} \begin{array}{r} 1 \\ 5 \end{array} \begin{array}{r} 0 \\ 2 \end{array} \begin{array}{r} 1 \\ 9 \end{array}$
3. $\begin{array}{r} 4 \\ 3 \end{array} \begin{array}{r} 3 \\ 6 \end{array} \begin{array}{r} 7 \\ 1 \end{array} \begin{array}{r} 5 \\ 5 \end{array} \begin{array}{r} 5 \\ 4 \end{array} \begin{array}{r} 0 \\ 1 \end{array} \begin{array}{r} 1 \\ 7 \end{array} \begin{array}{r} 9 \\ 1 \end{array} \begin{array}{r} 3 \\ 0 \end{array} \begin{array}{r} 7 \\ 0 \end{array}$
4. $\begin{array}{r} 0 \\ 8 \end{array} \begin{array}{r} 1 \\ 4 \end{array} \begin{array}{r} 4 \\ 4 \end{array} \begin{array}{r} 9 \\ 0 \end{array} \begin{array}{r} 2 \\ 9 \end{array} \begin{array}{r} 5 \\ 1 \end{array} \begin{array}{r} 2 \\ 5 \end{array} \begin{array}{r} 3 \\ 2 \end{array} \begin{array}{r} 6 \\ 2 \end{array} \begin{array}{r} 1 \\ 3 \end{array}$
5. $\begin{array}{r} 2 \\ 4 \end{array} \begin{array}{r} 4 \\ 5 \end{array} \begin{array}{r} 8 \\ 0 \end{array} \begin{array}{r} 4 \\ 2 \end{array} \begin{array}{r} 5 \\ 0 \end{array} \begin{array}{r} 4 \\ 0 \end{array} \begin{array}{r} 0 \\ 5 \end{array} \begin{array}{r} 7 \\ 7 \end{array} \begin{array}{r} 4 \\ 1 \end{array} \begin{array}{r} 8 \\ 8 \end{array}$
6. $\begin{array}{r} 2 \\ 6 \end{array} \begin{array}{r} 8 \\ 3 \end{array} \begin{array}{r} 2 \\ 3 \end{array} \begin{array}{r} 1 \\ 2 \end{array} \begin{array}{r} 0 \\ 6 \end{array} \begin{array}{r} 3 \\ 7 \end{array} \begin{array}{r} 7 \\ 2 \end{array} \begin{array}{r} 5 \\ 2 \end{array} \begin{array}{r} 9 \\ 2 \end{array} \begin{array}{r} 8 \\ 2 \end{array}$
7. $\begin{array}{r} 7 \\ 3 \end{array} \begin{array}{r} 3 \\ 4 \end{array} \begin{array}{r} 4 \\ 8 \end{array} \begin{array}{r} 6 \\ 5 \end{array} \begin{array}{r} 9 \\ 4 \end{array} \begin{array}{r} 6 \\ 6 \end{array} \begin{array}{r} 3 \\ 9 \end{array} \begin{array}{r} 6 \\ 4 \end{array} \begin{array}{r} 9 \\ 9 \end{array} \begin{array}{r} 5 \\ 6 \end{array}$
8. $\begin{array}{r} 3 \\ 5 \end{array} \begin{array}{r} 2 \\ 8 \end{array} \begin{array}{r} 6 \\ 7 \end{array} \begin{array}{r} 9 \\ 3 \end{array} \begin{array}{r} 6 \\ 3 \end{array} \begin{array}{r} 8 \\ 9 \end{array} \begin{array}{r} 7 \\ 6 \end{array} \begin{array}{r} 5 \\ 3 \end{array} \begin{array}{r} 7 \\ 4 \end{array} \begin{array}{r} 4 \\ 9 \end{array}$
9. $\begin{array}{r} 3 \\ 8 \end{array} \begin{array}{r} 2 \\ 7 \end{array} \begin{array}{r} 9 \\ 7 \end{array} \begin{array}{r} 7 \\ 5 \end{array} \begin{array}{r} 8 \\ 4 \end{array} \begin{array}{r} 6 \\ 9 \end{array} \begin{array}{r} 7 \\ 8 \end{array} \begin{array}{r} 6 \\ 8 \end{array} \begin{array}{r} 5 \\ 7 \end{array} \begin{array}{r} 9 \\ 6 \end{array}$
10. $\begin{array}{r} 4 \\ 6 \end{array} \begin{array}{r} 8 \\ 7 \end{array} \begin{array}{r} 7 \\ 9 \end{array} \begin{array}{r} 8 \\ 6 \end{array} \begin{array}{r} 4 \\ 7 \end{array} \begin{array}{r} 9 \\ 8 \end{array} \begin{array}{r} 5 \\ 8 \end{array} \begin{array}{r} 9 \\ 5 \end{array} \begin{array}{r} 5 \\ 9 \end{array} \begin{array}{r} 8 \\ 5 \end{array}$

I. Higher Decade Addition Facts

Say or write the sums.

1. $12+5$ $23+2$ $41+7$ $14+4$
2. $29+4$ $37+8$ $16+9$ $28+3$
3. $35+6$ $24+9$ $48+6$ $33+6$
4. $\begin{array}{r} 13 \\ 4 \end{array} \begin{array}{r} 11 \\ 5 \end{array} \begin{array}{r} 21 \\ 6 \end{array} \begin{array}{r} 36 \\ 2 \end{array} \begin{array}{r} 25 \\ 3 \end{array} \begin{array}{r} 27 \\ 1 \end{array} \begin{array}{r} 17 \\ 2 \end{array} \begin{array}{r} 20 \\ 4 \end{array}$
5. $\begin{array}{r} 22 \\ 8 \end{array} \begin{array}{r} 47 \\ 7 \end{array} \begin{array}{r} 11 \\ 9 \end{array} \begin{array}{r} 38 \\ 4 \end{array} \begin{array}{r} 19 \\ 7 \end{array} \begin{array}{r} 37 \\ 4 \end{array} \begin{array}{r} 34 \\ 8 \end{array} \begin{array}{r} 47 \\ 6 \end{array}$
6. $\begin{array}{r} 38 \\ 9 \end{array} \begin{array}{r} 18 \\ 9 \end{array} \begin{array}{r} 36 \\ 8 \end{array} \begin{array}{r} 27 \\ 5 \end{array} \begin{array}{r} 38 \\ 6 \end{array} \begin{array}{r} 15 \\ 7 \end{array} \begin{array}{r} 34 \\ 6 \end{array} \begin{array}{r} 26 \\ 9 \end{array}$

II. Single Column Addition

Write the sums on folded paper. Find the reason for any mistake you make. Practice so you will not make the same mistake again.

1. $\begin{array}{r} 3 \\ 9 \\ 5 \end{array} \begin{array}{r} 8 \\ 6 \\ 4 \end{array} \begin{array}{r} 6 \\ 5 \\ 5 \end{array} \begin{array}{r} 5 \\ 9 \\ 6 \end{array} \begin{array}{r} 3 \\ 2 \\ 9 \end{array} \begin{array}{r} 4 \\ 9 \\ 4 \end{array} \begin{array}{r} 9 \\ 6 \\ 8 \end{array} \begin{array}{r} 6 \\ 0 \\ 4 \end{array} \begin{array}{r} 4 \\ 7 \\ 5 \end{array} \begin{array}{r} 6 \\ 8 \\ 7 \end{array}$
2. $\begin{array}{r} 8 \\ 4 \\ 5 \\ 2 \\ 7 \\ 9 \end{array} \begin{array}{r} 9 \\ 9 \\ 5 \\ 2 \\ 3 \\ 3 \end{array} \begin{array}{r} 5 \\ 9 \\ 4 \\ 9 \\ 1 \\ 3 \end{array} \begin{array}{r} 3 \\ 6 \\ 2 \\ 3 \\ 5 \\ 4 \end{array} \begin{array}{r} 6 \\ 4 \\ 8 \\ 0 \\ 6 \\ 2 \end{array} \begin{array}{r} 8 \\ 9 \\ 3 \\ 5 \\ 3 \\ 8 \end{array} \begin{array}{r} 7 \\ 6 \\ 0 \\ 9 \\ 8 \\ 3 \end{array} \begin{array}{r} 5 \\ 4 \\ 8 \\ 2 \\ 6 \\ 3 \end{array} \begin{array}{r} 4 \\ 0 \\ 4 \\ 7 \\ 6 \\ 8 \end{array} \begin{array}{r} 9 \\ 1 \\ 8 \\ 3 \\ 9 \\ 2 \end{array}$

I. Carrying in Addition

Write the sums on folded paper. If you make a mistake, find it and learn not to make it again.

1.	$\begin{array}{r} 28 \\ 36 \\ \hline \end{array}$	$\begin{array}{r} 325 \\ 156 \\ \hline \end{array}$	$\begin{array}{r} 763 \\ 182 \\ \hline \end{array}$	$\begin{array}{r} 564 \\ 278 \\ \hline \end{array}$	$\begin{array}{r} 2345 \\ 1423 \\ \hline \end{array}$	$\begin{array}{r} 1054 \\ 2046 \\ \hline \end{array}$
2.	$\begin{array}{r} 54 \\ 27 \\ \hline \end{array}$	$\begin{array}{r} 438 \\ 249 \\ \hline \end{array}$	$\begin{array}{r} 554 \\ 275 \\ \hline \end{array}$	$\begin{array}{r} 857 \\ 97 \\ \hline \end{array}$	$\begin{array}{r} 3056 \\ 2428 \\ \hline \end{array}$	$\begin{array}{r} 2858 \\ 1142 \\ \hline \end{array}$
3.	$\begin{array}{r} 39 \\ 44 \\ \hline \end{array}$	$\begin{array}{r} 304 \\ 409 \\ \hline \end{array}$	$\begin{array}{r} 414 \\ 393 \\ \hline \end{array}$	$\begin{array}{r} 369 \\ 231 \\ \hline \end{array}$	$\begin{array}{r} 5408 \\ 1503 \\ \hline \end{array}$	$\begin{array}{r} 5532 \\ 1728 \\ \hline \end{array}$
4.	$\begin{array}{r} 78 \\ 63 \\ \hline \end{array}$	$\begin{array}{r} 507 \\ 938 \\ \hline \end{array}$	$\begin{array}{r} 282 \\ 97 \\ \hline \end{array}$	$\begin{array}{r} 546 \\ 295 \\ \hline \end{array}$	$\begin{array}{r} 3054 \\ 2949 \\ \hline \end{array}$	$\begin{array}{r} 3006 \\ 2908 \\ \hline \end{array}$

II. Column Addition

Write the sums on folded paper.

86	43	98	80	432	771	54
63	32	6	7	189	192	302
92	59	73	77	243	43	28
85	60	42	5	991	800	7
47	73	8	40	417	6	600

III. Column Addition with Copying

Copy neatly in straight columns and add.

- 43, 72, 90, 16, 37
- 50, 8, 73, 29, 4
- 275, 986, 513, 791, 804
- 543, 27, 117, 8, 940
- 2647, 957, 88, 502

The 100 Subtraction Facts

Find the subtraction facts that you miss. Practice on these until you know them. The few you do not know will make many answers wrong.

1.	$\begin{array}{r} 4 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 10 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 11 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 14 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 16 \\ 8 \\ \hline \end{array}$
2.	$\begin{array}{r} 2 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 0 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 10 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 11 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 13 \\ 4 \\ \hline \end{array}$
3.	$\begin{array}{r} 9 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 10 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 10 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 15 \\ 6 \\ \hline \end{array}$
4.	$\begin{array}{r} 2 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 11 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 10 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 11 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 13 \\ 9 \\ \hline \end{array}$
5.	$\begin{array}{r} 8 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 10 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 11 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 18 \\ 9 \\ \hline \end{array}$
6.	$\begin{array}{r} 9 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 10 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 17 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 13 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 11 \\ 7 \\ \hline \end{array}$
7.	$\begin{array}{r} 4 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 10 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 13 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 11 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 16 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ 7 \\ \hline \end{array}$
8.	$\begin{array}{r} 5 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 15 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 14 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 15 \\ 9 \\ \hline \end{array}$
9.	$\begin{array}{r} 5 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 10 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 15 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 16 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 14 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 11 \\ 4 \\ \hline \end{array}$
10.	$\begin{array}{r} 9 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 13 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 14 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 14 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 13 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 17 \\ 9 \\ \hline \end{array}$

I. Borrowing in Subtraction

Subtract. Check by adding minuend and subtrahend.

1.	$\begin{array}{r} 434 \\ 115 \\ \hline \end{array}$	$\begin{array}{r} 745 \\ 329 \\ \hline \end{array}$	$\begin{array}{r} 864 \\ 427 \\ \hline \end{array}$	$\begin{array}{r} 983 \\ 349 \\ \hline \end{array}$	$\begin{array}{r} 677 \\ 248 \\ \hline \end{array}$	$\begin{array}{r} 5691 \\ 2438 \\ \hline \end{array}$
2.	$\begin{array}{r} 328 \\ 146 \\ \hline \end{array}$	$\begin{array}{r} 642 \\ 281 \\ \hline \end{array}$	$\begin{array}{r} 735 \\ 262 \\ \hline \end{array}$	$\begin{array}{r} 549 \\ 194 \\ \hline \end{array}$	$\begin{array}{r} 847 \\ 582 \\ \hline \end{array}$	$\begin{array}{r} 4928 \\ 2545 \\ \hline \end{array}$
3.	$\begin{array}{r} 531 \\ 185 \\ \hline \end{array}$	$\begin{array}{r} 764 \\ 275 \\ \hline \end{array}$	$\begin{array}{r} 417 \\ 259 \\ \hline \end{array}$	$\begin{array}{r} 634 \\ 355 \\ \hline \end{array}$	$\begin{array}{r} 844 \\ 375 \\ \hline \end{array}$	$\begin{array}{r} 7344 \\ 4829 \\ \hline \end{array}$
4.	$\begin{array}{r} 175 \\ 83 \\ \hline \end{array}$	$\begin{array}{r} 134 \\ 67 \\ \hline \end{array}$	$\begin{array}{r} 783 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 919 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 724 \\ 84 \\ \hline \end{array}$	$\begin{array}{r} 1643 \\ 154 \\ \hline \end{array}$

II. Zeros in Subtraction

Subtract. Check by adding. When you make a mistake, find the reason for it and then practice so you will not make that mistake again.

1.	$\begin{array}{r} 460 \\ 214 \\ \hline \end{array}$	$\begin{array}{r} 504 \\ 352 \\ \hline \end{array}$	$\begin{array}{r} 906 \\ 628 \\ \hline \end{array}$	$\begin{array}{r} 720 \\ 143 \\ \hline \end{array}$	$\begin{array}{r} 800 \\ 425 \\ \hline \end{array}$	$\begin{array}{r} 6020 \\ 1432 \\ \hline \end{array}$
2.	$\begin{array}{r} 524 \\ 118 \\ \hline \end{array}$	$\begin{array}{r} 672 \\ 585 \\ \hline \end{array}$	$\begin{array}{r} 758 \\ 238 \\ \hline \end{array}$	$\begin{array}{r} 863 \\ 456 \\ \hline \end{array}$	$\begin{array}{r} 354 \\ 263 \\ \hline \end{array}$	$\begin{array}{r} 7143 \\ 2136 \\ \hline \end{array}$
3.	$\begin{array}{r} 305 \\ 106 \\ \hline \end{array}$	$\begin{array}{r} 700 \\ 143 \\ \hline \end{array}$	$\begin{array}{r} 403 \\ 308 \\ \hline \end{array}$	$\begin{array}{r} 603 \\ 209 \\ \hline \end{array}$	$\begin{array}{r} 800 \\ 205 \\ \hline \end{array}$	$\begin{array}{r} 9010 \\ 4004 \\ \hline \end{array}$

4. $80530 - 60604$ 6. $61506 - 20397$

5. $90003 - 80905$ 7. $50030 - 29103$

The 100 Multiplication Facts

Practice makes permanent. Correct practice makes perfect. Find the multiplication facts that cause you trouble, and practice on them until you know them.

1.	$\begin{array}{r} 2 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 0 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 1 \\ \hline \end{array}$
2.	$\begin{array}{r} 5 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ 4 \\ \hline \end{array}$
3.	$\begin{array}{r} 4 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ 3 \\ \hline \end{array}$
4.	$\begin{array}{r} 1 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 0 \\ 0 \\ \hline \end{array}$
5.	$\begin{array}{r} 8 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 0 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 0 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 0 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 0 \\ 2 \\ \hline \end{array}$
6.	$\begin{array}{r} 8 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 1 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 0 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 0 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 0 \\ \hline \end{array}$
7.	$\begin{array}{r} 3 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ 8 \\ \hline \end{array}$
8.	$\begin{array}{r} 8 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 0 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 8 \\ \hline \end{array}$
9.	$\begin{array}{r} 4 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 0 \\ \hline \end{array}$	$\begin{array}{r} 2 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 0 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 5 \\ \hline \end{array}$
10.	$\begin{array}{r} 5 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ 6 \\ \hline \end{array}$

I. Multiplying and Adding

Say the answers to a partner, or write them.

1. $8 \times 7 + 3$ $7 \times 9 + 1$ $7 \times 8 + 1$ $9 \times 7 + 8$
2. $6 \times 7 + 4$ $4 \times 0 + 3$ $6 \times 7 + 5$ $2 \times 8 + 1$
3. $5 \times 9 + 1$ $8 \times 7 + 4$ $7 \times 9 + 3$ $9 \times 8 + 8$
4. $8 \times 9 + 4$ $7 \times 6 + 2$ $8 \times 9 + 7$ $6 \times 0 + 2$
5. $9 \times 7 + 1$ $7 \times 8 + 3$ $9 \times 5 + 6$ $9 \times 9 + 6$
6. $9 \times 3 + 6$ $9 \times 7 + 4$ $8 \times 7 + 7$ $3 \times 9 + 2$
7. $7 \times 9 + 5$ $7 \times 7 + 5$ $9 \times 1 + 6$ $8 \times 9 + 3$
8. $4 \times 4 + 3$ $8 \times 9 + 6$ $8 \times 8 + 3$ $7 \times 6 + 6$
9. $7 \times 8 + 5$ $6 \times 7 + 2$ $9 \times 7 + 6$ $8 \times 7 + 6$
10. $7 \times 6 + 1$ $9 \times 8 + 7$ $7 \times 6 + 5$ $6 \times 7 + 3$
11. $9 \times 8 + 5$ $8 \times 2 + 1$ $8 \times 4 + 7$ $7 \times 8 + 6$
12. $8 \times 0 + 7$ $1 \times 9 + 0$ $9 \times 8 + 1$ $7 \times 9 + 6$
13. $8 \times 7 + 2$ $3 \times 4 + 1$ $5 \times 0 + 4$ $3 \times 9 + 0$

II. One-Figure Multipliers

Find the products. Check by covering your work and multiplying a second time.

1. $\begin{array}{r} 28 \\ 7 \end{array}$ $\begin{array}{r} 45 \\ 3 \end{array}$ $\begin{array}{r} 806 \\ 6 \end{array}$ $\begin{array}{r} 540 \\ 9 \end{array}$ $\begin{array}{r} 6073 \\ 8 \end{array}$ $\begin{array}{r} 40607 \\ 9 \end{array}$
2. $\begin{array}{r} 30 \\ 4 \end{array}$ $\begin{array}{r} 37 \\ 5 \end{array}$ $\begin{array}{r} 145 \\ 8 \end{array}$ $\begin{array}{r} 938 \\ 6 \end{array}$ $\begin{array}{r} 6592 \\ 7 \end{array}$ $\begin{array}{r} 73841 \\ 3 \end{array}$
3. $\begin{array}{r} 51 \\ 6 \end{array}$ $\begin{array}{r} 67 \\ 9 \end{array}$ $\begin{array}{r} 304 \\ 3 \end{array}$ $\begin{array}{r} 775 \\ 8 \end{array}$ $\begin{array}{r} 3754 \\ 4 \end{array}$ $\begin{array}{r} 21605 \\ 6 \end{array}$
4. $\begin{array}{r} 24 \\ 9 \end{array}$ $\begin{array}{r} 36 \\ 8 \end{array}$ $\begin{array}{r} 500 \\ 3 \end{array}$ $\begin{array}{r} 908 \\ 7 \end{array}$ $\begin{array}{r} 8005 \\ 6 \end{array}$ $\begin{array}{r} 20505 \\ 8 \end{array}$

I. Two-Figure and Three-Figure Multipliers

Multiply. Check by multiplying a second time.

1. $\begin{array}{r} 26 \\ 53 \end{array}$ $\begin{array}{r} 91 \\ 39 \end{array}$ $\begin{array}{r} 452 \\ 41 \end{array}$ $\begin{array}{r} 764 \\ 86 \end{array}$ $\begin{array}{r} 932 \\ 92 \end{array}$ $\begin{array}{r} 8623 \\ 75 \end{array}$
2. $\begin{array}{r} 54 \\ 47 \end{array}$ $\begin{array}{r} 83 \\ 16 \end{array}$ $\begin{array}{r} 538 \\ 627 \end{array}$ $\begin{array}{r} 917 \\ 234 \end{array}$ $\begin{array}{r} 669 \\ 358 \end{array}$ $\begin{array}{r} 6483 \\ 967 \end{array}$
3. $\begin{array}{r} 37 \\ 58 \end{array}$ $\begin{array}{r} 19 \\ 73 \end{array}$ $\begin{array}{r} 267 \\ 93 \end{array}$ $\begin{array}{r} 546 \\ 287 \end{array}$ $\begin{array}{r} 679 \\ 46 \end{array}$ $\begin{array}{r} 2864 \\ 398 \end{array}$

II. Multipliers that End in 0

Copy and multiply.

1. 20×198 300×685 4000×284
2. 70×564 500×247 6000×329
3. 60×386 700×394 8000×915
4. 90×470 100×760 7000×600

III. Zeros in Multiplier or Multiplicand

Multiply. Check by going over your work.

1. $\begin{array}{r} 206 \\ 43 \end{array}$ $\begin{array}{r} 709 \\ 28 \end{array}$ $\begin{array}{r} 507 \\ 39 \end{array}$ $\begin{array}{r} 6108 \\ 54 \end{array}$ $\begin{array}{r} 94030 \\ 67 \end{array}$
2. $\begin{array}{r} 340 \\ 63 \end{array}$ $\begin{array}{r} 850 \\ 78 \end{array}$ $\begin{array}{r} 400 \\ 87 \end{array}$ $\begin{array}{r} 5040 \\ 34 \end{array}$ $\begin{array}{r} 76009 \\ 56 \end{array}$
3. $\begin{array}{r} 387 \\ 206 \end{array}$ $\begin{array}{r} 542 \\ 604 \end{array}$ $\begin{array}{r} 604 \\ 409 \end{array}$ $\begin{array}{r} 8705 \\ 308 \end{array}$ $\begin{array}{r} 92006 \\ 4007 \end{array}$

The 90 Division Facts

Say these quotients to a partner or write them on folded paper. When writing quotients on folded paper, hold the paper above the row of exercises.

1. $4\overline{)8}$ $1\overline{)8}$ $3\overline{)15}$ $6\overline{)24}$ $2\overline{)10}$ $4\overline{)24}$
2. $6\overline{)0}$ $1\overline{)6}$ $9\overline{)63}$ $3\overline{)27}$ $5\overline{)20}$ $8\overline{)40}$
3. $2\overline{)0}$ $1\overline{)2}$ $5\overline{)45}$ $6\overline{)54}$ $7\overline{)42}$ $7\overline{)35}$
4. $1\overline{)4}$ $3\overline{)9}$ $8\overline{)72}$ $9\overline{)45}$ $7\overline{)56}$ $4\overline{)32}$
5. $6\overline{)6}$ $9\overline{)9}$ $7\overline{)49}$ $8\overline{)32}$ $7\overline{)28}$ $9\overline{)81}$
6. $3\overline{)0}$ $3\overline{)3}$ $6\overline{)48}$ $9\overline{)27}$ $7\overline{)63}$ $5\overline{)35}$
7. $4\overline{)4}$ $8\overline{)8}$ $6\overline{)42}$ $5\overline{)40}$ $9\overline{)72}$ $6\overline{)36}$
8. $8\overline{)0}$ $9\overline{)0}$ $8\overline{)64}$ $6\overline{)30}$ $4\overline{)36}$ $8\overline{)56}$
9. $1\overline{)9}$ $2\overline{)4}$ $9\overline{)36}$ $5\overline{)25}$ $9\overline{)54}$ $4\overline{)28}$
10. $2\overline{)6}$ $1\overline{)3}$ $7\overline{)21}$ $8\overline{)48}$ $5\overline{)30}$ $3\overline{)24}$
11. $3\overline{)6}$ $1\overline{)7}$ $3\overline{)12}$ $2\overline{)18}$ $2\overline{)14}$ $3\overline{)21}$
12. $5\overline{)5}$ $5\overline{)0}$ $4\overline{)12}$ $8\overline{)24}$ $5\overline{)15}$ $4\overline{)20}$
13. $1\overline{)1}$ $7\overline{)7}$ $7\overline{)14}$ $6\overline{)12}$ $2\overline{)12}$ $9\overline{)18}$
14. $1\overline{)5}$ $1\overline{)0}$ $2\overline{)16}$ $3\overline{)18}$ $8\overline{)16}$ $2\overline{)8}$
15. $2\overline{)2}$ $4\overline{)0}$ $7\overline{)0}$ $6\overline{)18}$ $4\overline{)16}$ $5\overline{)10}$

I. Uneven Division Facts

Find the quotients and remainders.

1. $4\overline{)39}$ $8\overline{)79}$ $9\overline{)67}$ $8\overline{)68}$ $9\overline{)49}$
2. $8\overline{)61}$ $6\overline{)40}$ $8\overline{)75}$ $6\overline{)47}$ $9\overline{)39}$
3. $7\overline{)45}$ $9\overline{)85}$ $7\overline{)51}$ $9\overline{)74}$ $6\overline{)45}$
4. $4\overline{)38}$ $6\overline{)55}$ $7\overline{)59}$ $7\overline{)67}$ $8\overline{)50}$
5. $9\overline{)52}$ $7\overline{)54}$ $6\overline{)37}$ $8\overline{)59}$ $8\overline{)51}$
6. $8\overline{)60}$ $7\overline{)68}$ $9\overline{)56}$ $9\overline{)71}$ $7\overline{)53}$
7. $9\overline{)75}$ $9\overline{)78}$ $7\overline{)47}$ $7\overline{)62}$ $9\overline{)44}$
8. $8\overline{)52}$ $7\overline{)58}$ $5\overline{)47}$ $9\overline{)57}$ $6\overline{)51}$
9. $8\overline{)76}$ $9\overline{)58}$ $7\overline{)61}$ $5\overline{)48}$ $8\overline{)66}$
10. $7\overline{)48}$ $7\overline{)46}$ $8\overline{)77}$ $6\overline{)56}$ $9\overline{)43}$

II. One-Figure Divisors

Divide. Place each quotient figure correctly. Check by multiplying.

1. $2\overline{)7854}$ $3\overline{)8634}$ $4\overline{)7832}$ $5\overline{)9385}$
2. $6\overline{)9474}$ $7\overline{)6783}$ $8\overline{)9496}$ $9\overline{)8298}$
3. $2\overline{)4156}$ $3\overline{)7524}$ $4\overline{)7052}$ $5\overline{)8020}$
4. $6\overline{)2448}$ $7\overline{)2317}$ $8\overline{)6520}$ $9\overline{)4320}$
5. $2\overline{)1320}$ $3\overline{)2721}$ $4\overline{)3004}$ $5\overline{)1510}$
6. $6\overline{)4902}$ $7\overline{)2114}$ $8\overline{)8016}$ $9\overline{)1188}$

I. Easy Division with Two-Figure Divisors

Divide. Check by multiplying.

- | | | | |
|------------------------|----------------------|-----------------------|-----------------------|
| 1. $46 \overline{)92}$ | $85 \overline{)510}$ | $51 \overline{)2448}$ | $62 \overline{)3472}$ |
| 2. $29 \overline{)58}$ | $59 \overline{)295}$ | $92 \overline{)5796}$ | $33 \overline{)2079}$ |
| 3. $38 \overline{)76}$ | $88 \overline{)792}$ | $73 \overline{)4161}$ | $54 \overline{)2970}$ |
| 4. $42 \overline{)84}$ | $93 \overline{)651}$ | $84 \overline{)5964}$ | $55 \overline{)3465}$ |
| 5. $24 \overline{)72}$ | $98 \overline{)784}$ | $46 \overline{)1472}$ | $66 \overline{)4026}$ |
| 6. $34 \overline{)68}$ | $79 \overline{)237}$ | $37 \overline{)1221}$ | $77 \overline{)4158}$ |
| 7. $53 \overline{)86}$ | $77 \overline{)693}$ | $48 \overline{)1968}$ | $98 \overline{)5292}$ |
| 8. $61 \overline{)74}$ | $69 \overline{)220}$ | $69 \overline{)1587}$ | $89 \overline{)5429}$ |

II. Harder Division with Two-Figure Divisors

Divide. Test each quotient figure.

- | | | | |
|--------------------------|-----------------------|-----------------------|------------------------|
| 1. $26 \overline{)1912}$ | $74 \overline{)4662}$ | $63 \overline{)4795}$ | $48 \overline{)35608}$ |
| 2. $84 \overline{)4546}$ | $46 \overline{)3404}$ | $58 \overline{)4814}$ | $73 \overline{)41619}$ |
| 3. $58 \overline{)2378}$ | $44 \overline{)3640}$ | $47 \overline{)3854}$ | $87 \overline{)45247}$ |
| 4. $53 \overline{)2491}$ | $72 \overline{)3672}$ | $77 \overline{)4158}$ | $98 \overline{)43365}$ |
| 5. $28 \overline{)2436}$ | $84 \overline{)6556}$ | $63 \overline{)7654}$ | $76 \overline{)45828}$ |
| 6. $42 \overline{)4326}$ | $43 \overline{)4171}$ | $36 \overline{)7380}$ | $93 \overline{)18972}$ |
| 7. $14 \overline{)8526}$ | $17 \overline{)3863}$ | $19 \overline{)5890}$ | $34 \overline{)20638}$ |
| 8. $13 \overline{)6635}$ | $16 \overline{)6632}$ | $18 \overline{)9018}$ | $28 \overline{)25284}$ |

I. Rounding Numbers

1. Round each number to the nearest—

- Ten** 673 416 882 207 735
Hundred 874 847 650 309 6138
Thousand 76,805 8,386 459,641
Million 63,548,216 9,215,756 9,823,250

Round to the nearest **thousand** and add.

2. 98,640; 8,544; 1,238; 35,478; 19,675
 3. 175,348; 47,500; 28,108; 83,675; 42,089

Round to the nearest **million**. Subtract.

4. $37,278,156 - 8,640,738$
 5. $163,803,396 - 96,292,301$

Round to the nearest **hundred**. Multiply.

6. $8,487 \times 3,532$ 7. $26,325 \times 9,667$

Round to the nearest **thousand**. Divide.

8. $93,512,823 \div 3,123$ 9. $1,639,846 \div 3,917$

II. Estimating with Rounded Numbers

No pencils. Round each number as you think best. Name the best answer from among those in parentheses. Check.

- | | |
|------------------------|----------------------|
| 1. $21 + 47 + 94 + 38$ | (180; 200; 210) |
| 2. $469 - 158$ | (300; 310; 320) |
| 3. $48 \times \$7.20$ | (\$28; \$35; \$350) |
| 4. $\$746.10 \div 75$ | (\$8; \$9; \$10) |
| 5. $111,131 + 27,688$ | (138,000; 139,000) |
| 6. $78,164 - 5,328$ | (73,000; 25,000) |
| 7. 23×873 | (1600; 1800; 18,000) |
| 8. $71,816 \div 312$ | (24; 240; 24,000) |

I. Reducing Fractions to Lowest Terms

Reduce each fraction to lowest terms.

1. $\frac{2}{4}$ $\frac{3}{12}$ $\frac{2}{12}$ $\frac{4}{10}$ $\frac{14}{16}$ $\frac{4}{16}$ $\frac{8}{10}$ $\frac{15}{18}$
2. $\frac{16}{24}$ $\frac{4}{20}$ $\frac{3}{6}$ $\frac{24}{36}$ $\frac{8}{24}$ $\frac{4}{8}$ $\frac{20}{32}$ $\frac{3}{9}$
3. $\frac{2}{10}$ $\frac{6}{12}$ $\frac{6}{16}$ $\frac{2}{8}$ $\frac{4}{12}$ $\frac{5}{10}$ $\frac{9}{12}$ $\frac{15}{20}$
4. $\frac{4}{6}$ $\frac{5}{20}$ $\frac{3}{18}$ $\frac{2}{16}$ $\frac{6}{24}$ $\frac{8}{32}$ $\frac{6}{8}$ $\frac{8}{16}$
5. $\frac{4}{32}$ $\frac{10}{16}$ $\frac{6}{9}$ $\frac{6}{10}$ $\frac{12}{16}$ $\frac{8}{12}$ $\frac{10}{20}$ $\frac{10}{12}$

II. Changing Fractions to Higher Terms

Supply the missing numbers.

1. $\frac{1}{2} = \frac{?}{4}$ $\frac{3}{4} = \frac{?}{8}$ $\frac{2}{3} = \frac{?}{6}$ $\frac{1}{4} = \frac{?}{8}$ $\frac{1}{3} = \frac{?}{18}$
2. $\frac{1}{5} = \frac{?}{10}$ $\frac{1}{3} = \frac{?}{6}$ $\frac{2}{5} = \frac{?}{10}$ $\frac{1}{2} = \frac{?}{12}$ $\frac{1}{5} = \frac{?}{15}$
3. $\frac{1}{4} = \frac{?}{32}$ $\frac{3}{5} = \frac{?}{20}$ $\frac{1}{2} = \frac{?}{8}$ $\frac{2}{3} = \frac{?}{15}$ $\frac{3}{4} = \frac{?}{12}$
4. $\frac{1}{3} = \frac{?}{12}$ $\frac{1}{4} = \frac{?}{20}$ $\frac{5}{6} = \frac{?}{18}$ $\frac{3}{4} = \frac{?}{24}$ $\frac{1}{8} = \frac{?}{32}$
5. $\frac{1}{6} = \frac{?}{12}$ $\frac{1}{2} = \frac{?}{10}$ $\frac{3}{4} = \frac{?}{16}$ $\frac{1}{3} = \frac{?}{9}$ $\frac{1}{2} = \frac{?}{6}$
6. $\frac{1}{2} = \frac{?}{16}$ $\frac{2}{3} = \frac{?}{12}$ $\frac{5}{8} = \frac{?}{16}$ $\frac{2}{5} = \frac{?}{25}$ $\frac{5}{6} = \frac{?}{24}$
7. $\frac{1}{5} = \frac{?}{20}$ $\frac{3}{8} = \frac{?}{24}$ $\frac{1}{4} = \frac{?}{16}$ $\frac{7}{8} = \frac{?}{32}$ $\frac{1}{4} = \frac{?}{24}$

III. Finding the Least Common Denominator

Find the L. C. D. of each group of fractions.

1. $\frac{1}{2}, \frac{1}{4}$ $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ $\frac{5}{12}, \frac{2}{3}, \frac{1}{4}$ $\frac{2}{5}, \frac{2}{3}, \frac{1}{2}$
2. $\frac{1}{2}, \frac{1}{3}$ $\frac{1}{2}, \frac{1}{3}, \frac{5}{6}$ $\frac{1}{6}, \frac{3}{4}, \frac{2}{3}$ $\frac{7}{8}, \frac{3}{5}, \frac{3}{4}$
3. $\frac{3}{4}, \frac{5}{8}$ $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$ $\frac{7}{12}, \frac{3}{8}, \frac{1}{4}$ $\frac{5}{6}, \frac{1}{15}, \frac{2}{3}$
4. $\frac{1}{3}, \frac{3}{4}$ $\frac{3}{8}, \frac{1}{6}, \frac{2}{3}$ $\frac{5}{12}, \frac{1}{2}, \frac{2}{3}$ $\frac{2}{15}, \frac{2}{3}, \frac{3}{5}$

Improper Fractions, Mixed Numbers, and Whole Numbers

I. Change each of the following to a mixed number or a whole number. Reduce any fraction in answer to lowest terms.

1. $\frac{3}{2}$ $\frac{25}{3}$ $\frac{6}{3}$ $\frac{12}{8}$ $\frac{10}{4}$ $\frac{20}{15}$ $\frac{9}{6}$ $\frac{12}{6}$
2. $\frac{20}{16}$ $\frac{8}{5}$ $\frac{25}{8}$ $\frac{4}{2}$ $\frac{16}{6}$ $\frac{6}{4}$ $\frac{12}{3}$ $\frac{12}{5}$
3. $\frac{5}{4}$ $\frac{4}{3}$ $\frac{8}{6}$ $\frac{22}{5}$ $\frac{10}{3}$ $\frac{7}{2}$ $\frac{10}{6}$ $\frac{8}{4}$
4. $\frac{15}{5}$ $\frac{16}{8}$ $\frac{12}{4}$ $\frac{10}{8}$ $\frac{30}{8}$ $\frac{15}{3}$ $\frac{20}{5}$ $\frac{8}{2}$

II. Change each of the following to an improper fraction.

1. $1\frac{2}{3}$ $1\frac{3}{4}$ $12\frac{1}{2}$ $2\frac{1}{2}$ $1\frac{2}{5}$ $7\frac{1}{2}$ $2\frac{1}{6}$ $6\frac{7}{8}$
2. $3\frac{3}{4}$ $2\frac{5}{6}$ $3\frac{4}{5}$ $1\frac{5}{6}$ $3\frac{5}{6}$ $1\frac{7}{8}$ $3\frac{1}{2}$ $4\frac{3}{8}$
3. $4\frac{1}{6}$ $8\frac{1}{3}$ $1\frac{5}{8}$ $3\frac{1}{8}$ $6\frac{1}{4}$ $3\frac{1}{3}$ $2\frac{5}{8}$ $6\frac{2}{3}$

III. Copy, supplying each missing number.

Additions

1. $2\frac{1}{2} = 2\frac{?}{12}$ $3\frac{3}{4} = 3\frac{?}{12}$ $5\frac{1}{6} = 5\frac{?}{12}$ $10\frac{17}{12}$ or $11\frac{5}{12}$
2. $1\frac{7}{8} = 1\frac{?}{24}$ $3\frac{1}{3} = 3\frac{?}{24}$ $4\frac{5}{6} = 4\frac{?}{24}$ $8\frac{?}{24}$ or $\frac{?}{24}$
3. $6\frac{2}{3} = 6\frac{?}{12}$ $1\frac{3}{4} = 1\frac{?}{12}$ $7\frac{1}{2} = 7\frac{?}{12}$ $14\frac{?}{12}$ or $\frac{?}{12}$

Subtractions

4. $6\frac{1}{4} = 5\frac{5}{4}$ $3\frac{3}{4} = 3\frac{3}{4}$ $2\frac{?}{4}$ or $2\frac{1}{2}$
5. $4\frac{1}{6} = 3\frac{?}{6}$ $1\frac{5}{6} = 1\frac{5}{6}$ $2\frac{?}{6}$ or $\frac{?}{3}$
6. $7\frac{1}{2} = 6\frac{?}{8}$ $2\frac{7}{8} = 2\frac{7}{8}$ $4\frac{?}{8}$

Adding Fractions and Mixed Numbers

Add. Leave each sum as a whole number or as a fraction or mixed number in lowest terms.

$$1. \begin{array}{r} \frac{1}{2} \\ \frac{1}{2} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{3} \\ \frac{1}{3} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{4} \\ \frac{1}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{6} \\ \frac{1}{6} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{8} \\ \frac{1}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{2}{5} \\ \frac{3}{5} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{4} \\ \frac{1}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{8} \\ \frac{5}{8} \\ \hline \end{array}$$

$$2. \begin{array}{r} \frac{1}{2} \\ \frac{1}{3} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{2} \\ \frac{1}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{3} \\ \frac{1}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{4} \\ \frac{2}{3} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{2} \\ \frac{3}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{3} \\ \frac{1}{6} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{2} \\ \frac{1}{6} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{4} \\ \frac{1}{8} \\ \hline \end{array}$$

$$3. \begin{array}{r} \frac{2}{3} \\ \frac{1}{2} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{4} \\ \frac{1}{3} \\ \hline \end{array} \quad \begin{array}{r} \frac{5}{6} \\ \frac{3}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{4} \\ \frac{7}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{5}{6} \\ \frac{7}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{5}{6} \\ \frac{2}{3} \\ \hline \end{array} \quad \begin{array}{r} \frac{5}{8} \\ \frac{3}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{9}{16} \\ \frac{3}{4} \\ \hline \end{array}$$

$$4. \begin{array}{r} \frac{7}{12} \\ \frac{2}{3} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{4} \\ \frac{5}{12} \\ \hline \end{array} \quad \begin{array}{r} \frac{7}{16} \\ \frac{7}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{2}{3} \\ \frac{11}{15} \\ \hline \end{array} \quad \begin{array}{r} \frac{5}{8} \\ \frac{5}{12} \\ \hline \end{array} \quad \begin{array}{r} \frac{5}{9} \\ \frac{2}{3} \\ \hline \end{array} \quad \begin{array}{r} \frac{5}{6} \\ \frac{4}{9} \\ \hline \end{array} \quad \begin{array}{r} \frac{7}{12} \\ \frac{8}{9} \\ \hline \end{array}$$

$$5. \begin{array}{r} \frac{3}{8} \\ \frac{2}{3} \\ \frac{3}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{5}{6} \\ \frac{3}{5} \\ \frac{1}{2} \\ \hline \end{array} \quad \begin{array}{r} \frac{4}{5} \\ \frac{2}{3} \\ \frac{7}{10} \\ \hline \end{array} \quad \begin{array}{r} \frac{5}{12} \\ \frac{3}{4} \\ \frac{2}{3} \\ \hline \end{array} \quad \begin{array}{r} \frac{7}{12} \\ \frac{7}{8} \\ \frac{1}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{5}{6} \\ \frac{3}{4} \\ \frac{2}{3} \\ \hline \end{array} \quad \begin{array}{r} \frac{8}{15} \\ \frac{2}{3} \\ \frac{3}{5} \\ \hline \end{array} \quad \begin{array}{r} \frac{7}{8} \\ \frac{5}{6} \\ \frac{1}{2} \\ \hline \end{array}$$

$$6. \begin{array}{r} 2\frac{2}{3} \\ 3\frac{1}{3} \\ \hline \end{array} \quad \begin{array}{r} 6\frac{1}{4} \\ 2\frac{1}{2} \\ \hline \end{array} \quad \begin{array}{r} 7\frac{1}{2} \\ 3\frac{3}{4} \\ \hline \end{array} \quad \begin{array}{r} 8\frac{1}{3} \\ 2\frac{1}{6} \\ \hline \end{array} \quad \begin{array}{r} 6\frac{2}{3} \\ 2\frac{1}{2} \\ \hline \end{array} \quad \begin{array}{r} 6\frac{3}{4} \\ 6\frac{2}{3} \\ \hline \end{array}$$

$$7. \begin{array}{r} 2\frac{1}{2} \\ 3\frac{1}{3} \\ 6\frac{1}{4} \\ \hline \end{array} \quad \begin{array}{r} 6\frac{2}{3} \\ 2\frac{5}{8} \\ 3\frac{3}{4} \\ \hline \end{array} \quad \begin{array}{r} 2\frac{3}{5} \\ 4\frac{1}{2} \\ 1\frac{7}{10} \\ \hline \end{array} \quad \begin{array}{r} 7\frac{1}{2} \\ 3\frac{4}{5} \\ 6\frac{1}{3} \\ \hline \end{array} \quad \begin{array}{r} 8\frac{5}{6} \\ 2\frac{2}{3} \\ 3\frac{3}{4} \\ \hline \end{array} \quad \begin{array}{r} 6\frac{7}{8} \\ 3\frac{1}{3} \\ 1\frac{1}{6} \\ \hline \end{array}$$

$$8. 3\frac{1}{3} + 2\frac{1}{2} + 4 + 1\frac{3}{4} + 1\frac{7}{8}$$

$$9. 6\frac{1}{4} + 9 + 2\frac{3}{4} + 1\frac{7}{16} + 3\frac{7}{8} + 1\frac{1}{2}$$

Subtracting Fractions and Mixed Numbers

Subtract. Leave each remainder as a whole number or as a fraction or mixed number in lowest terms. Check by addition.

$$1. \begin{array}{r} \frac{2}{3} \\ \frac{1}{3} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{4} \\ \frac{1}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{5}{6} \\ \frac{1}{6} \\ \hline \end{array} \quad \begin{array}{r} \frac{7}{8} \\ \frac{3}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{7}{8} \\ \frac{5}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{7}{12} \\ \frac{3}{12} \\ \hline \end{array} \quad \begin{array}{r} \frac{9}{16} \\ \frac{5}{16} \\ \hline \end{array} \quad \begin{array}{r} \frac{15}{16} \\ \frac{3}{16} \\ \hline \end{array}$$

$$2. \begin{array}{r} \frac{2}{3} \\ \frac{1}{2} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{4} \\ \frac{1}{2} \\ \hline \end{array} \quad \begin{array}{r} \frac{7}{8} \\ \frac{1}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{5}{6} \\ \frac{1}{3} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{4} \\ \frac{1}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{5}{6} \\ \frac{1}{2} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{4} \\ \frac{1}{6} \\ \hline \end{array} \quad \begin{array}{r} \frac{7}{8} \\ \frac{2}{3} \\ \hline \end{array}$$

$$3. 1\frac{3}{4} - \frac{1}{2} = 2\frac{1}{2} - \frac{1}{4} = 3\frac{1}{3} - \frac{2}{3} = 6\frac{1}{4} - \frac{3}{4} = 2\frac{1}{8} - \frac{3}{8} = 6\frac{1}{6} - \frac{5}{6}$$

$$4. 8\frac{1}{3} - 6\frac{2}{3} = 7\frac{1}{2} - 3\frac{3}{4} = 6\frac{1}{4} - 2\frac{1}{2} = 4\frac{1}{6} - 2\frac{5}{6} = 2\frac{1}{8} - 1\frac{5}{8} = 3\frac{3}{8} - 1\frac{3}{4}$$

$$5. 4 - 1\frac{3}{4} = 8 - 3\frac{1}{3} = 9 - 7\frac{1}{2} = 8\frac{1}{3} - 6 = 3\frac{3}{4} - 2 = 2\frac{1}{2} - 1$$

$$6. 18\frac{3}{4} - 9\frac{1}{2} = 16\frac{2}{3} - 3\frac{3}{4} = 17\frac{1}{2} - 1\frac{3}{4} = 25 - 8\frac{1}{3} = 32\frac{1}{2} - 17\frac{1}{2} = 37\frac{1}{2} - 16\frac{2}{3}$$

$$7. 10\frac{4}{5} - 3\frac{1}{5} = 12\frac{1}{2} - 3\frac{3}{5} = 16\frac{2}{3} - 4\frac{3}{5} = 27\frac{1}{2} - 18\frac{3}{4} = 20\frac{5}{6} - 13\frac{1}{3} = 15\frac{1}{6} - 8\frac{2}{3}$$

$$8. 1\frac{3}{16} - \frac{7}{16} = 4\frac{7}{12} - 1\frac{5}{12} = 6\frac{4}{15} - 2\frac{3}{5} = 7\frac{3}{10} - 5\frac{1}{2} = 7\frac{9}{16} - 4\frac{3}{16} = 8\frac{7}{8} - 3\frac{7}{12}$$

I. Multiplying Fractions and Mixed Numbers

Multiply. Cancel when you can.

1. $\frac{1}{2} \times 6$ $\frac{2}{3} \times 12$ $\frac{1}{3} \times 15$ $\frac{1}{4} \times 8$ $\frac{3}{4} \times 12$
2. $\frac{2}{3} \times \frac{3}{4}$ $\frac{1}{2} \times \frac{4}{15}$ $\frac{1}{3} \times \frac{3}{8}$ $\frac{5}{6} \times \frac{3}{5}$ $\frac{1}{6} \times \frac{3}{8}$
3. $\frac{5}{8} \times 16$ $\frac{2}{3} \times 6$ $\frac{7}{8} \times 24$ $\frac{3}{4} \times 20$ $\frac{5}{6} \times 24$
4. $\frac{1}{2} \times \frac{8}{15}$ $\frac{2}{3} \times \frac{4}{9}$ $\frac{2}{3} \times \frac{9}{16}$ $\frac{3}{4} \times \frac{9}{16}$ $\frac{1}{6} \times \frac{4}{5}$
5. $6 \times 1\frac{1}{2}$ $8 \times \frac{3}{4}$ $12 \times 1\frac{1}{3}$ $15 \times 2\frac{2}{3}$ $18 \times 2\frac{5}{6}$
6. $2\frac{1}{2} \times 4$ $3\frac{1}{3} \times 6$ $6\frac{2}{3} \times 12$ $6\frac{3}{4} \times 16$ $12\frac{1}{2} \times 8$
7. $42 \times 8\frac{1}{3}$ $8 \times 17\frac{1}{2}$ $7 \times 14\frac{2}{7}$ $9 \times 16\frac{2}{3}$ $8 \times 18\frac{3}{4}$
8. $16\frac{2}{3} \times 4$ $32\frac{1}{2} \times 5$ $8\frac{3}{4} \times 22$ $8\frac{1}{3} \times 25$ $3\frac{7}{8} \times 50$

II. Dividing Fractions and Mixed Numbers

Divide. Leave each quotient in best form.

1. $4 \div \frac{1}{2}$ $6 \div \frac{1}{3}$ $8 \div \frac{1}{4}$ $10 \div \frac{5}{6}$ $12 \div \frac{3}{4}$
2. $\frac{3}{4} \div \frac{7}{8}$ $\frac{5}{6} \div \frac{5}{8}$ $\frac{2}{3} \div \frac{4}{15}$ $\frac{7}{8} \div \frac{3}{4}$ $\frac{9}{16} \div \frac{3}{4}$
3. $\frac{1}{2} \div 4$ $\frac{2}{3} \div 2$ $\frac{3}{4} \div 4$ $\frac{7}{8} \div 2$ $\frac{5}{6} \div 3$
4. $1\frac{1}{2} \div 3$ $3\frac{1}{3} \div 5$ $7\frac{1}{2} \div 2$ $6\frac{2}{3} \div 4$ $8\frac{3}{4} \div 5$
5. $6 \div 1\frac{1}{2}$ $9 \div 2\frac{1}{4}$ $15 \div 3\frac{3}{4}$ $50 \div 8\frac{1}{3}$ $15 \div 3\frac{1}{3}$
6. $7\frac{1}{2} \div \frac{3}{4}$ $8\frac{1}{3} \div \frac{2}{3}$ $4\frac{1}{6} \div \frac{4}{5}$ $3\frac{1}{2} \div \frac{7}{8}$
7. $17\frac{1}{2} \div 2\frac{1}{2}$ $6\frac{3}{4} \div 1\frac{7}{8}$ $16\frac{2}{3} \div 4\frac{1}{6}$ $5\frac{5}{6} \div 1\frac{3}{4}$
8. $1\frac{7}{8} \div 7\frac{1}{2}$ $3\frac{1}{2} \div 2\frac{2}{3}$ $3\frac{3}{8} \div 6\frac{3}{4}$ $8\frac{1}{3} \div 3\frac{1}{3}$
9. $6\frac{2}{3} \div \frac{5}{6}$ $6\frac{7}{8} \div 2\frac{3}{4}$ $7\frac{1}{2} \div 6\frac{2}{3}$ $4\frac{1}{5} \div 1\frac{1}{2}$

I. Addition of Decimals

Find the sums.

1.
$$\begin{array}{r} 3.3 \\ 4.0 \\ \hline \end{array}$$

$$\begin{array}{r} 9.87 \\ .36 \\ \hline \end{array}$$

$$\begin{array}{r} 7.134 \\ 6.09 \\ \hline \end{array}$$

$$\begin{array}{r} 3.643 \\ .846 \\ \hline 97.390 \end{array}$$

$$\begin{array}{r} 7.184 \\ 28.9 \\ \hline .63 \end{array}$$
2.
$$\begin{array}{r} 9.1 \\ .8 \\ 3.2 \\ .9 \\ 3.5 \\ 1.8 \\ .1 \\ \hline \end{array}$$

$$\begin{array}{r} 5.3 \\ 4.0 \\ .6 \\ 2.9 \\ .8 \\ 3.0 \\ 1.4 \\ \hline \end{array}$$

$$\begin{array}{r} .5 \\ 3.9 \\ 1.6 \\ .5 \\ 41.0 \\ 8.0 \\ 3.2 \\ \hline \end{array}$$

$$\begin{array}{r} 1.35 \\ .64 \\ .09 \\ 6.84 \\ .38 \\ 1.56 \\ .40 \\ \hline \end{array}$$

$$\begin{array}{r} .4 \\ .07 \\ 1.6 \\ .759 \\ .3 \\ 6.47 \\ .005 \\ \hline \end{array}$$

3. $1.65 + .05 + 21.7$
4. $.376 + .5 + 172.1$
5. $7.18 + 16 + .043$
6. $756.1 + 19 + .02$

II. Subtraction of Decimals

Subtract.

1.
$$\begin{array}{r} 9.1 \\ 3.3 \\ \hline \end{array}$$

$$\begin{array}{r} 33.5 \\ 15.7 \\ \hline \end{array}$$

$$\begin{array}{r} 81.60 \\ 7.35 \\ \hline \end{array}$$

$$\begin{array}{r} 312.11 \\ 90.28 \\ \hline \end{array}$$

$$\begin{array}{r} 5.3014 \\ 3.8554 \\ \hline \end{array}$$
2.
$$\begin{array}{r} 3.2 \\ .5 \\ \hline \end{array}$$

$$\begin{array}{r} 6.32 \\ 1.45 \\ \hline \end{array}$$

$$\begin{array}{r} 7.302 \\ 6.595 \\ \hline \end{array}$$

$$\begin{array}{r} 42.005 \\ 3.907 \\ \hline \end{array}$$

$$\begin{array}{r} 40.003 \\ 35.009 \\ \hline \end{array}$$
3.
$$\begin{array}{r} 1.0 \\ .1 \\ \hline \end{array}$$

$$\begin{array}{r} 4.5 \\ 3.75 \\ \hline \end{array}$$

$$\begin{array}{r} 41.30 \\ 8.17 \\ \hline \end{array}$$

$$\begin{array}{r} 6.11 \\ 2.0005 \\ \hline \end{array}$$

$$\begin{array}{r} 16.3 \\ 9.678 \\ \hline \end{array}$$

4. $9.4 - 1.673$
5. $234.18 - 97.4$
6. $32.1 - 29.108$
7. $14.4 - 7.2834$

I. Multiplication of Decimals

Copy with the decimal point placed correctly in the products.

1. $2.8 \times 7 = 196$
2. $.39 \times .6 = 234$
3. $3.76 \times .04 = 1504$
4. $83.32 \times .04 = 33328$
5. $.008 \times 4.6 = 368$
6. $.0072 \times 12 = 864$
7. $9.63 \times .37 = 35631$
8. $.093 \times .8 = 744$

Multiply.

- | | | | | |
|---|--|---|---|--|
| 9. $\begin{array}{r} 9.7 \\ .6 \\ \hline \end{array}$ | $\begin{array}{r} 8.54 \\ .8 \\ \hline \end{array}$ | $\begin{array}{r} 7.183 \\ 7 \\ \hline \end{array}$ | $\begin{array}{r} 47.342 \\ .4 \\ \hline \end{array}$ | $\begin{array}{r} 42.286 \\ 9 \\ \hline \end{array}$ |
| 10. $\begin{array}{r} 8.3 \\ .08 \\ \hline \end{array}$ | $\begin{array}{r} 85.6 \\ .56 \\ \hline \end{array}$ | $\begin{array}{r} 67.34 \\ 8.9 \\ \hline \end{array}$ | $\begin{array}{r} 6.1342 \\ 17 \\ \hline \end{array}$ | $\begin{array}{r} 654.76 \\ 7.2 \\ \hline \end{array}$ |

II. Division of Decimals

Copy with decimal point placed correctly in the quotients.

1. $253.5 \div 1.5 = 169$
2. $46.72 \div .73 = 64$
3. $19.712 \div 70.4 = 28$
4. $1.9388 \div .037 = 524$
5. $.7266 \div 17.3 = 42$
6. $617.12 \div 30.4 = 203$
7. $165.76 \div .32 = 518$
8. $3.9474 \div .0043 = 918$

Divide.

- | | | |
|--------------------------------|----------------------------|----------------------------|
| 9. $4.3 \overline{)187.61}$ | $.34 \overline{)1.8088}$ | $.047 \overline{)296.57}$ |
| 10. $.23 \overline{)1048.8}$ | $50.3 \overline{)3615.6}$ | $.724 \overline{)1375.6}$ |
| 11. $3.75 \overline{)1012.5}$ | $.021 \overline{)2083.2}$ | $72.7 \overline{)40.712}$ |
| 12. $.0043 \overline{)2.5499}$ | $.607 \overline{)183.314}$ | $.0147 \overline{)37.926}$ |

Parts of 100

I. Change each of these common fractions or mixed numbers to a—

a. decimal as hundredths with any remainder as a common fraction, as $\frac{1}{7} = .14\frac{2}{7}$.

b. per cent.

1. $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{6}$ $\frac{5}{6}$ $\frac{1}{8}$ $\frac{3}{8}$
2. $\frac{5}{8}$ $\frac{7}{8}$ $\frac{1}{10}$ $\frac{3}{10}$ $\frac{7}{10}$ $\frac{9}{10}$ $\frac{1}{12}$ $\frac{5}{12}$ $\frac{7}{12}$
3. $\frac{11}{12}$ $\frac{1}{16}$ $\frac{3}{16}$ $\frac{5}{16}$ $\frac{15}{16}$ $\frac{1}{15}$ $\frac{2}{15}$ $\frac{1}{24}$ $\frac{1}{32}$
4. $1\frac{1}{2}$ $2\frac{3}{4}$ $3\frac{1}{3}$ $6\frac{2}{3}$ $4\frac{1}{6}$ $2\frac{1}{4}$ $2\frac{1}{6}$

II. Change each of these decimals to a—

a. common fraction or mixed number in lowest terms.

b. per cent.

1. .05 .04 .10 .20 .30 .40 .50 .75
2. $.08\frac{1}{3}$ $.16\frac{2}{3}$ $.33\frac{1}{3}$ $.66\frac{2}{3}$ $.06\frac{2}{3}$ $.03\frac{1}{3}$ $.01\frac{2}{3}$ $.04\frac{1}{6}$
3. $.12\frac{1}{2}$ $.37\frac{1}{2}$ $.62\frac{1}{2}$ $.87\frac{1}{2}$ $.06\frac{1}{4}$ $.02\frac{1}{2}$ $.07\frac{1}{2}$ $.03\frac{3}{4}$
4. 1.20 2.25 3.75 4.00 $2.66\frac{2}{3}$ $1.16\frac{2}{3}$

III. Change each of these per cents to a—

a. common fraction or mixed number in lowest terms.

b. decimal.

1. 4% 6% 10% 20% 75% 50%
2. $33\frac{1}{3}\%$ $66\frac{2}{3}\%$ $16\frac{2}{3}\%$ $8\frac{1}{3}\%$ $6\frac{2}{3}\%$ $3\frac{1}{3}\%$
3. $6\frac{1}{4}\%$ $12\frac{1}{2}\%$ 25% $87\frac{1}{2}\%$ $3\frac{3}{4}\%$ $37\frac{1}{2}\%$
4. 175% 110% 104% 225% $\frac{1}{2}\%$ $\frac{1}{5}\%$

The Three Types of Percentage

One partner will work Exercises 1-20 while the other works Exercises 21-40. They will then exchange and mark papers and work out together any exercise that either missed.

1. 6 is $\frac{1}{2}\%$ of 12.
2. 4 is 20% of $\frac{1}{2}$.
3. $\frac{1}{2}$ is 10% of 50.
4. 4 is $\frac{1}{2}\%$ of 24.
5. 2 is $6\frac{1}{4}\%$ of $\frac{1}{2}$.
6. $\frac{1}{2}$ is $33\frac{1}{3}\%$ of 24.
7. 12 is $\frac{1}{2}\%$ of 4.
8. 16 is 200% of $\frac{1}{2}$.
9. $\frac{1}{2}$ is $\frac{1}{4}\%$ of 400.
10. 18 is $\frac{1}{2}\%$ of 900.
11. 9 is $\frac{1}{2}\%$ of 12.
12. 3 is 60% of $\frac{1}{2}$.
13. $\frac{1}{2}$ is 15% of 40.
14. 3 is $\frac{1}{2}\%$ of 36.
15. 3 is $12\frac{1}{2}\%$ of $\frac{1}{2}$.
16. $\frac{1}{2}$ is $6\frac{2}{3}\%$ of 45.
17. 2 is $\frac{1}{2}\%$ of 400.
18. 3 is $\frac{1}{2}\%$ of $\frac{1}{2}$.
19. $\frac{1}{2}$ is 300% of 6.
20. 6 is $\frac{1}{2}\%$ of 4.
21. 12 is 3% of $\frac{1}{2}$.
22. $\frac{1}{2}$ is $16\frac{2}{3}\%$ of 24.
23. $\frac{1}{2}$ is $8\frac{1}{3}\%$ of 36.
24. 18 is $\frac{1}{2}\%$ of 9.
25. $\frac{1}{2}$ is $6\frac{1}{4}\%$ of 640.
26. 12 is 300% of $\frac{1}{2}$.
27. 180 is $\frac{1}{2}\%$ of 90.
28. $\frac{1}{2}$ is $16\frac{2}{3}\%$ of 2400.
29. 120 is 30% of $\frac{1}{2}$.
30. 3 is $\frac{1}{2}\%$ of 600.
31. 180 is $\frac{1}{2}\%$ of 9.
32. 12 is 30% of $\frac{1}{2}$.
33. 180 is $\frac{1}{2}\%$ of 900.
34. $\frac{1}{2}$ is $6\frac{1}{4}\%$ of 64.
35. 120 is 3% of $\frac{1}{2}$.
36. $\frac{1}{2}$ is $6\frac{1}{4}\%$ of 4800.
37. 18 is $\frac{1}{2}\%$ of 90.
38. $\frac{1}{2}$ is $16\frac{2}{3}\%$ of 240.
39. 18 is $\frac{1}{2}\%$ of 9000.
40. 120 is 300% of $\frac{1}{2}$.

Recognizing Types of Percentage

In percentage exercises it is very important that you be wholly sure of the process needed to find the right answer. If you know and use the right process, getting the right answer will be easy and sure. Go over the exercises on page 336. Tell the process you use in each and why you use it.

The three processes used in solving percentage exercises are given in the formulas below:

$$\text{Part} = \text{rate} \times \text{whole}$$

$$\text{Rate} = \text{part} \div \text{whole}$$

$$\text{Whole} = \text{part} \div \text{rate}$$

Tell the process and find the answer for each exercise below.

1. 128 is $\frac{1}{2}\%$ of 192.
2. 64% of $\frac{1}{2}$ is 80.
3. 32 is $8\frac{1}{3}\%$ of $\frac{1}{2}$.
4. 72% of $\frac{1}{2}$ is 18.
5. $62\frac{1}{2}\%$ of $\frac{1}{2}$ is 50.
6. 120 is $166\frac{2}{3}\%$ of $\frac{1}{2}$.
7. $\frac{3}{4}\%$ of $\frac{1}{2}$ is 9.
8. 21 is 84% of $\frac{1}{2}$.
9. $7\frac{1}{2}\%$ of $\frac{1}{2}$ is 9.
10. $\frac{1}{2}$ is 150% of 48.
11. 84 is $133\frac{1}{3}\%$ of $\frac{1}{2}$.
12. $66\frac{2}{3}\%$ of 600 is $\frac{1}{2}$.
13. $\frac{1}{2}\%$ of 1000 is 5.
14. $\frac{1}{2}\%$ of 50 is 32.
15. $\frac{5}{8}\%$ of $\frac{1}{2}$ is 50.
16. 192 is $\frac{1}{2}\%$ of 128.
17. $\frac{1}{2}$ is $6\frac{2}{3}\%$ of 300.
18. $\frac{1}{2}$ is 125% of 60.
19. $\frac{2}{3}\%$ of 600 is $\frac{1}{2}$.
20. 32 is $\frac{1}{2}\%$ of 256.

I. Adding Denominate Numbers

Add. Leave each answer in simplest form.

1. $\begin{array}{r} 5 \text{ lb. } 2 \text{ oz.} \\ 2 \text{ lb. } 8 \text{ oz.} \end{array}$ $\begin{array}{r} 3 \text{ gal. } 1 \text{ qt.} \\ 2 \text{ gal. } 2 \text{ qt.} \end{array}$ $\begin{array}{r} 3 \text{ yd. } 1 \text{ ft. } 7 \text{ in.} \\ 2 \text{ yd. } 1 \text{ ft. } 2 \text{ in.} \end{array}$
2. $\begin{array}{r} 5 \text{ bu. } 3 \text{ pk.} \\ 2 \text{ bu. } 2 \text{ pk.} \end{array}$ $\begin{array}{r} 6 \text{ lb. } 8 \text{ oz.} \\ 2 \text{ lb. } 9 \text{ oz.} \end{array}$ $\begin{array}{r} 2 \text{ yd. } 2 \text{ ft. } 8 \text{ in.} \\ 3 \text{ yd. } 2 \text{ ft. } 6 \text{ in.} \end{array}$
3. $\begin{array}{r} 2 \text{ min. } 45 \text{ sec.} \\ 1 \text{ min. } 30 \text{ sec.} \end{array}$ $\begin{array}{r} 3 \text{ qt. } 1 \text{ pt.} \\ 2 \text{ qt. } 1 \text{ pt.} \end{array}$ $\begin{array}{r} 2 \text{ bu. } 2 \text{ pk. } 5 \text{ qt.} \\ 1 \text{ bu. } 3 \text{ pk. } 6 \text{ qt.} \end{array}$
4. $\begin{array}{r} 5 \text{ mo. } 18 \text{ da.} \\ 2 \text{ mo. } 24 \text{ da.} \end{array}$ $\begin{array}{r} 2 \text{ wk. } 4 \text{ da.} \\ 1 \text{ wk. } 5 \text{ da.} \end{array}$ $\begin{array}{r} 4 \text{ hr. } 32 \text{ min. } 30 \text{ sec.} \\ 1 \text{ hr. } 40 \text{ min. } 48 \text{ sec.} \end{array}$

II. Subtracting Denominate Numbers

Subtract. Check by adding.

1. $\begin{array}{r} 6 \text{ lb. } 8 \text{ oz.} \\ 2 \text{ lb. } 12 \text{ oz.} \end{array}$ $\begin{array}{r} 6 \text{ yd. } 1 \text{ ft.} \\ 1 \text{ yd. } 2 \text{ ft.} \end{array}$ $\begin{array}{r} 6 \text{ hr. } 18 \text{ min. } 20 \text{ sec.} \\ 2 \text{ hr. } 30 \text{ min. } 10 \text{ sec.} \end{array}$
2. $\begin{array}{r} 5 \text{ ft. } 4 \text{ in.} \\ 8 \text{ in.} \end{array}$ $\begin{array}{r} 3 \text{ hr.} \\ 40 \text{ min.} \end{array}$ $\begin{array}{r} 5 \text{ bu. } 2 \text{ pk. } 1 \text{ qt.} \\ 3 \text{ bu. } 1 \text{ pk. } 3 \text{ qt.} \end{array}$
3. $\begin{array}{r} 3 \text{ gal. } 1 \text{ qt.} \\ 1 \text{ gal. } 3 \text{ qt.} \end{array}$ $\begin{array}{r} 1 \text{ yd. } 1 \text{ ft.} \\ 2 \text{ ft.} \end{array}$ $\begin{array}{r} 3 \text{ yd. } 1 \text{ ft. } 5 \text{ in.} \\ 1 \text{ yd. } 2 \text{ ft. } 9 \text{ in.} \end{array}$
4. $\begin{array}{r} 4 \text{ ft. } 6 \text{ in.} \\ 3 \text{ ft. } 8 \text{ in.} \end{array}$ $\begin{array}{r} 3 \text{ wk. } 3 \text{ da.} \\ 1 \text{ wk. } 6 \text{ da.} \end{array}$ $\begin{array}{r} 12 \text{ mi. } 160 \text{ rd.} \\ 6 \text{ mi. } 300 \text{ rd.} \end{array}$

I. Changing to Smaller or Larger Units of Measure

Say or write the number that belongs in place of each blank.

1. 5 qt. 1 pt. = __ pt. 11. 11 pt. = __ qt. __ pt.
2. 5 ft. 8 in. = __ in. 12. 27 in. = __ ft. __ in.
3. 2 yd. 1 ft. = __ ft. 13. 16 ft. = __ yd. __ ft.
4. 3 lb. 10 oz. = __ oz. 14. 50 oz. = __ lb. __ oz.
5. 2 hr. 5 min. = __ min. 15. 200 min. = __ hr. __ min.
6. 5 min. 10 sec. = __ sec. 16. 60 hr. = __ da. __ hr.
7. 2 wk. 3 da. = __ da. 17. 22 pk. = __ bu. __ pk.
8. 1 bu. 3 pk. = __ pk. 18. 18 qt. = __ gal. __ qt.
9. 2 gal. 2 qt. = __ qt. 19. 50 da. = __ mo. __ da.
10. 2 mo. 15 da. = __ da. 20. 25 da. = __ wk. __ da.

II. Finding the Time Between Dates

Find the number of years, months, and days—

1. from Nov. 11, 1918, till December 7, 1941
2. from July 16, 1943, till October 19, 1947
3. from August 4, 1947, till December 15, 1951
4. from September 3, 1946, till January 5, 1951

Find the exact number of days from—

5. April 3 till next July 1
6. December 9 till next February 25
7. May 31 till next October 28
8. August 17 till next November 25
9. June 14 till next August 12
10. March 8 till next August 15

I. Multiplication of Denominate Numbers

Multiply. Leave products in simplest form.

$$\begin{array}{r} 1. \quad 2 \text{ ft. } 3 \text{ in.} \quad 8 \text{ lb. } 5 \text{ oz.} \quad 5 \text{ yd. } 2 \text{ ft.} \\ \quad \quad \quad \underline{3} \quad \quad \quad \underline{6} \quad \quad \quad \underline{7} \end{array}$$

$$\begin{array}{r} 2. \quad 5 \text{ min. } 9 \text{ sec.} \quad 3 \text{ bu. } 2 \text{ pk.} \quad 2 \text{ gal. } 3 \text{ qt.} \\ \quad \quad \quad \underline{8} \quad \quad \quad \underline{5} \quad \quad \quad \underline{6} \end{array}$$

Find the area of each of these rectangles in two ways:

a. Reduce feet to inches before multiplying.

b. Change inches to fractions of foot before multiplying.

3. $l = 2 \text{ ft. } 3 \text{ in.}; w = 8 \text{ in.}$

4. $l = 3 \text{ ft. } 4 \text{ in.}; w = 2 \text{ ft. } 3 \text{ in.}$

5. $l = 7 \text{ ft. } 6 \text{ in.}; w = 1 \text{ ft. } 4 \text{ in.}$

6. $l = 13 \text{ ft. } 4 \text{ in.}; w = 3 \text{ ft. } 9 \text{ in.}$

II. Division of Denominate Numbers

Divide.

$$1. \quad \overline{4)8 \text{ lb. } 12 \text{ oz.}} \quad \overline{3)6 \text{ ft. } 9 \text{ in.}} \quad \overline{5)10 \text{ hr. } 40 \text{ min.}}$$

$$2. \quad \overline{2)3 \text{ yd. } 1 \text{ ft.}} \quad \overline{3)5 \text{ gal. } 1 \text{ qt.}} \quad \overline{6)8 \text{ mo. } 24 \text{ da.}}$$

$$3. \quad \overline{6)7 \text{ bu. } 2 \text{ pk.}} \quad \overline{8)9 \text{ lb. } 8 \text{ oz.}} \quad \overline{9)5 \text{ yr. } 3 \text{ mo.}}$$

$$4. \quad \overline{5)13 \text{ ft. } 4 \text{ in.}} \quad \overline{7)16 \text{ yd. } 1 \text{ ft.}} \quad \overline{6)7 \text{ gal. } 2 \text{ qt.}}$$

$$5. \quad \overline{8)18 \text{ sq. ft. } 32 \text{ sq. in.}} \quad \overline{9)22 \text{ sq. ft. } 36 \text{ sq. in.}}$$

$$6. \quad \overline{6)15 \text{ sq. yd. } 3 \text{ sq. ft.}} \quad \overline{7)17 \text{ sq. yd. } 1 \text{ sq. ft.}}$$

